

LEE COUNTY MATH INVITATIONAL
hosted by FORT MYERS HIGH SCHOOL
MARCH 14, 1998

CALCULUS TEAM 1.

Find the relative minimum of $y = f(x) = x^4 + x^3 + \frac{x^2}{2} + 1$ on $[-3, 2]$

CALCULUS TEAM 2.

An ellipse with horizontal semi-major axis length 3 and semi-minor axis length 2 has center at the midpoint of the line segment with endpoints (4, 7) and (-2, 1). What is the slope of the tangent line to the ellipse at the point $\left(2, \frac{12 + 4\sqrt{2}}{3}\right)$

CALCULUS TEAM 3.

Find the derivative of $x^4 + 3x^2 + 2$ with respect to $x^3 + 1$ at $x = 3$.

CALCULUS TEAM 4.

$$\int_{-3}^3 \left[(x^4 + x^2 + 1)(x^7 - x) \left(\cos \frac{\pi x}{3} \right) + (x - 1)^2 (2x - 2) + 2 \right] dx =$$

CALCULUS TEAM 5.

$$A = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{1}{k^2 + k} \right)$$

$$B = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{1}{2} \right)^k$$

$$C = \lim_{n \rightarrow \infty} \sum_{k=0}^n \frac{\left(\frac{1}{2} \right)^k}{k!}$$

find $(A - B + C)^{2\pi i}$ where $i = \sqrt{-1}$

CALCULUS TEAM 6.

A = the volume of the solid resulting when the region between $y = -x^2 + 4$ and the x-axis is revolved about the x-axis.

B = the volume of the solid resulting when the region between $y = x^2$ and $y = x + 2$ is revolved about the x-axis.

C = the volume of the solid resulting when the region between $y = x^3$ and $y = x^6$ is revolved about the x-axis.

Find $\frac{AB}{64C}$

CALCULUS TEAM 7.

John loves eating Twinkies. He has eaten so many that his body is now of a perfect spherical shape. John eats 200 Twinkies a minute (each at a volume of 10 cubic cm.). At what rate does the radius of his body increase when his total volume is 36π cubic m? Express the answer in cm/min.

CALCULUS TEAM 8.

A = area of the region enclosed by $r = \cos \theta$

B = area of the region enclosed by $r = \sin \theta$

C = area of the region enclosed by $r = \sin \theta + \cos \theta$

find $A + B + C$

CALCULUS TEAM 9.

Mike has a lot of hair. The amount of hair on Mike's body is given by $y = e^{6t} + 733$ where y is the amount of hair and t is the time in non-leap years. Mike is officially a monkey when the rate of his hair growth is 4 times his actual amount of hair. In how many days will Mike be officially a monkey? Round to the nearest tenth.

CALCULUS TEAM 10.

For $y = x^{x^{x^x}}$, find $\frac{y'}{y}$ at $y = 2$. Give the exact answer.

CALCULUS TEAM 11.

Find the positive number whose square root most exceeds its square.

CALCULUS TEAM 12

At what value of t , $t \geq 0$, is the velocity of a particle whose position is given by

$P(t) = \frac{1}{16}t^2 + 3t + \ln(t+1)$ a minimum? Exact answer, please.

CALCULUS TEAM 13.

Evaluate $\int_{-1}^1 \frac{(x^2 + x + 1)}{1 + x^2} dx$

CALCULUS TEAM 14.

For $y = 4x^4 - x^2 + 2$, find the larger of the x values of the inflection points of the function.

CALCULUS TEAM 15.

Approximate $\sqrt{83}$ using differentials.