Team Round March Regional Answers:

1. 12

3: $(x-3)^2 = [(-1)(x-3)]^2$
3: True for all cases. Proof by induction would give formal proof.
2: True when $x = 0$.
2: True everywhere except when $x = 0$.
2: True only when $x = 0$ *and* $x = 1$

2. $-34\frac{5}{7}$

Column 1	Column 2	Column 3
$5\log_8(6B-10) = 10$ $\log_8(6B-10) = 2$ $64 = 6B - 10$ $74 = 6B$ $B = \frac{37}{3}$	$9E^2 - 51E = -30$ $3(3E^2 - 17E + 10) = 0$ $3(3E-2)(E-5) = 0$ $E = \frac{2}{3}\ or\ 5$	$4(-17-3A)^{\frac{2}{3}} - 9 = 55$ $(-17-3A)^{\frac{2}{3}} = 16$ $-17 - 3A = \pm 64$ $A = -27\ or\ \frac{47}{3}$
$\frac{1}{4}\left\|2 - \frac{1}{5}F\right\| - \frac{10}{7} = -\frac{13}{14}$ $7\left\|2 - \frac{1}{5}F\right\| - 40 = -26$ $7\left\|2 - \frac{1}{5}F\right\| = 14$ $\left\|2 - \frac{1}{5}F\right\| = 2$ $2 - \frac{1}{5}F = \pm 2$ $F = 0\ or\ 20$	$-6(A-8) = 40 - 8A$ $-6A + 48 = 40 - 8A$ $2A = -8$ $A = -4$	$-2(5I-8)^{\frac{2}{5}} = -8$ $(5I-8)^{\frac{2}{5}} = 4$ $5I - 8 = \pm 32$ $I = 8\ or\ -\frac{24}{5}$
$V - 5 - 2V - 3 = -7V + 2V$ $-V - 8 = -5V$ $4V = 8$ $V = 2$	$\log_{15}(L^2 + 8L) = \log_{15}(6 + 3L)$ $L^2 + 8L = 6 + 3L$ $L^2 + 5L - 6 = 0$ $(L+6)(L-1) = 0$ $L = 1$ ($L = -6$ is extraneous).	$U^3 + 6U^2 + 5U = 0$ $U(U+5)(U+1) = 0$ $U = 0\ or\ U = -5\ or\ U = -1$

Column 4	Column 5
$\frac{4}{N^2-16} = \frac{4N^2 - 7N + 3}{N^2 - 16} - \frac{1}{N+4}$ $4 = 4N^2 - 7N + 3 - 1(N-4)$ $0 = 4N^2 - 8N + 3$ $0 = (2N-3)(2N-1)$ $N = \frac{3}{2}\ or\ \frac{1}{2}$	$\left(\frac{1}{625}\right)^{3A} = 25$ $25^{-6A} = 25^1$ $-6A = 1$ $A = -\frac{1}{6}$
$55L^2 + 28L - 16 = -5L^2$ $60L^2 + 28L - 16 = 0$	$T^3 - 3T = -2T^2 + 6$ $T^3 + 2T^2 - 3T - 6 = 0$

$(5L+4)(3L-1)=0$ $L=-\frac{4}{5}\ or\ \frac{1}{3}$	$(T+2)(T^2-3)=0$ $T=-2\ or \pm\sqrt{3}$
$\frac{\lvert 7S\rvert}{4}=3$ $\lvert 7S\rvert=12$ $S=\pm\frac{12}{7}$	$125^{3-2S}=625$ $5^{3(3-2S)}=5^4$ $9-6S=4$ $6S=5$ $S=\frac{5}{6}$

$$0+(-4)+(-27)+\left(-\frac{12}{7}\right)+(-2)=-33-\frac{12}{7}=-34\frac{5}{7}$$

3. 20

$(3x+2)^3=27x^3+54x^2+36x+8$
$(-2x-1)^4=16x^4+32x^3+24x^2+8x+1$
$\left(2x-\frac{1}{x}\right)^5=32x^5-80x^3+80x-\frac{40}{x}+\frac{10}{x^3}-\frac{1}{x^5}$
$((x+1)^2+1)^2=(x+1)^4+2(x+1)^2+1=x^4+4x^3+8x^2+8x+4$
$32x^5-17x^4-89x^3+22x^2+100x+3-\frac{40}{x}+\frac{10}{x^3}-\frac{1}{x^5}$

4. $(A\circ S\circ E\circ R)(2)=30$

$R(2)=\frac{1}{2}$
$E\left(\frac{1}{2}\right)=14$
$S(14)=-5$
$A(-5)=30$

5. 11

$A=64$
$B=1$
$C=4$
$D=8$
$2^x=ABCD=2^6 2^0 2^2 2^3=2^{11}$

6. $1-i$

$A=i^{-34}=\frac{1}{i^{34}}=\frac{1}{(i^2)^{17}}=-1$, (The remainder upon synthetic division is -34).
$B=i^{450\cdot 449\cdot 448\cdot\ldots\cdot 1}=(i^{448})^{450\cdot 449\cdot 447\cdot 446\cdot\ldots\cdot 1}=1$

$C=i^{200(3+799)/2}=i^{100(802)}=(i^{100})^{802}=1$, (Using the arithmetic series formula, we found $\frac{100(3+799)}{2}$.
$D=i^{(2398143)(2398141)}=(i^{2398143})^{2398141}=((-1)(i))^{2398141}=-i$
So $A+B+C+D=1-i$.

7. $-9-3i$

$x^2+18=(x\pm 3\sqrt{2}i)$	$x^3+8x=x(x\pm 2\sqrt{2}i)$	$x^4+5x^2+4=$ $(x\pm 2i)(x\pm i)$
$x^2+3ix-2=(x+2i)(x+i)$	$x^4-16=$ $(x\pm 2)(x\pm 2i)$	$x^4+26x^2+144=$ $(x\pm 3\sqrt{2}i)(x\pm 2\sqrt{2}i)$

So all that remains is $x(x-i)(x+2)(x-2)(x+2i)$. Evaluating this for $x=1$, we get $-3(1-i)(1+2i)=-3(3+i)$

The sum of the coefficients is -3 +3i

8. $(-32, -16)$

$$A = 4$$
$$B = \frac{1}{8}$$
$$C = 7$$
$$D = 8$$

So our transformed function is $4f\left(\frac{1}{8}x + 7\right) + 8$, which should be rewritten as $4f\left[\frac{1}{8}(x + 56)\right] + 8$. So all x values would be multiplied by 8 and then subtracted by 56. That would give -32. All the y-coordinates would be multiplied by 4 and added by eight, which would give us -16.

9. 44

$y = \sqrt{\lvert x \rvert}$ Even: 2 $y = \sqrt{\lvert(-x)\rvert}$ The y value will be the same for both, so it is even. It is also not odd, since y doesn't have a negative value.	$y = x^3 + x + 1$ Neither: 5 Using a quick power test for polynomials, we find that this is neither odd nor even, since both types of powers are represented.	$y = \frac{\lvert x \rvert}{x}$ Odd: 3 $y = \frac{\lvert(-x)\rvert}{-x} = -\frac{\lvert x \rvert}{x}$
$y = 10^x \log(x)$ Neither: 5 The domain doesn't even extend to the negative axis, so this is definitely neither.	$y = x^3 + \sqrt{x^2}$ Neither: 5 With the cubic involved, this cannot be even. However, checking for the odd side of things, the second term comes out positive every time, though it would need to come out negative to satisfy the negative definition.	$y = \pm\sqrt{1 - \frac{1}{x^2}}$ Both: 7 Putting in a negative x value creates a positive one. But having to solve for y, we see that we get both a positive and negative value, meeting the conditions for odd and even at the same time.
$x = y^3$ Odd: 3 $\sqrt[3]{(-x)} = y$ $\sqrt[3]{-1}\sqrt[3]{x} = y$ $-1\sqrt[3]{x} = y$	$x^2 + 4y^2 = 36$ Both: 7 Putting in a negative x value creates a positive one. But having to solve for y, we see that we get both a positive and negative value, meeting the conditions for odd and even at the same time.	$y = 0$ Both: 7 This is a horizontal line at zero. It is symmetric to the origin as well as symmetric to the y-axis.

10. $(-3, -4)$

Plugging in 0 for x, we find that the intercept is at $(0, -6), A = -6$

Factoring the expression, we get $\frac{(x+3)(x-2)(x+2)}{(x-2)(x-1)}$, so the removable discontinuity is located at $(2,20), B = 2, C = 10$

Filling in the system, we get $\begin{cases} -6x + 2y = 10 \\ 2x - 3y = 6 \end{cases}$. Using elimination, we find that the solution is at $(-3, -4)$.

11. $(0,2)$

A. Solving for the absolute value, we are given two equations to solve:

$x^2 - 5x + 10 < 0$ However, solving this further, it is clear that there are no such values for this expression that will yield a negative number. Therefore, the solutions will not come from this equation.	$0 > x^2 - x - 2$ $0 > (x-2)(x+1)$ Due to this, we can test a point between -1 and 2 and find that this interval is indeed negative. Therefore, the interval would be (-1,2)

B is best approached algebraically. $y = (x-2)^3 + 2(x-2)^2 + (x-2) + 2 = (x-2)^2(x-2+2) + 1(x-2+2) = x((x-2)^2+1)$. Setting this equal to zero, we see there is only one real root at $x = 0$. Based on end behavior, this graph tends to positive infinity, so it is positive from $(0, \infty)$ Therefore, the intersection of these two graphs is $(0,2)$.

12. 179

$(F \circ U \circ N \circ C \circ T \circ I \circ O \circ N)(2) = 65$
$(P \circ A \circ R \circ A \circ B \circ O \circ L \circ A)(2) = 15$
$(N \circ O \circ N \circ Z \circ E \circ R \circ O)(2) = 99$

13. 8335

Creating this sequence in a clearer form, we see that we get the sequence 1, 3, 7, 15, 31, … This is the sequence of Mersenne Primes. So a_n is a nearly geometric equation of the form $2^n - 1$. We can sum up the terms necessary:

$$31 + 63 + 127 + 255 + 511 + 1023 + 2047 + 4095 =$$

Oh wait. This is practically 2^n, except one short every time. So we can sum 2^n over this range and then subtract 8 from the final answer due to overcounting. $\frac{32(1-2^8)}{(1-2)} = -32(-255) = 8160 - 8 = 8152$.

For part B, we can start writing out the sequence. 1, 1, 1, 0, -1, -4, -6, -11, -11, -15, -4, -1, 36, 46, 121…

So we will need to sum these by hand: $-15 - 4 - 1 + 36 + 46 + 121 = 183$

$183 + 8152 = 8335$

14. -1568

Rewriting the equation in standard form, we get:

$(x+4)^2 + (y-8)^2 = -31 + 16 + 64 = 49$

So the radius is 7 and the center is located at $(-4,8)$.

$-4(49)(8) = -1568$

15. 460

$$0\begin{bmatrix} 0 & 2 & 7 \\ -3 & 0 & -5 \\ 7 & 3 & 0 \end{bmatrix} + 2\begin{bmatrix} 4 & 2 & 7 \\ -2 & 0 & -5 \\ 4 & 3 & 0 \end{bmatrix} + 4\begin{bmatrix} 4 & 0 & 7 \\ -2 & -3 & -5 \\ 4 & 7 & 0 \end{bmatrix} + 0\begin{bmatrix} 4 & 0 & 2 \\ -2 & -3 & 0 \\ 4 & 7 & 3 \end{bmatrix}$$
$0 + 2[0 - 40 - 42 - 0 + 60 - 0] + 4[0 + 0 - 98 + 84 + 140 + 0] + 0$
$2[-22] + 4[126] = 2[-22 + 252] = 460$