Question 1

Solve the following, where $i = \sqrt{-1}$:

A: $\frac{3+i}{1+3i}$

B: $\frac{(1+i)^9}{i^{39}}$

C: $det\begin{pmatrix} 2i & 5 & -3i & -4 \\ 1 & -i & 2 & 1 \\ 0 & 0 & -3 & i \\ 0 & 0 & 4i & 2 \end{pmatrix}$

Find $10A + B + C$.

Buchholz January Statewide 2018 Precalculus Team Round

Question 1

Solve the following, where $i = \sqrt{-1}$:

A: $\frac{3+i}{1+3i}$

B: $\frac{(1+i)^9}{i^{39}}$

C: $det\begin{pmatrix} 2i & 5 & -3i & -4 \\ 1 & -i & 2 & 1 \\ 0 & 0 & -3 & i \\ 0 & 0 & 4i & 2 \end{pmatrix}$

Find $10A + B + C$.

Question 2

Consider the conic given by $9x^2 + 25y^2 - 54x + 100y = 44$. Let $Ax + By = C$ be the line with negative slope going through the point with maximal y-value and a focus, where A, B, and C are relatively prime and $A > 0$. Let D be the length of the latus rectum.

Consider another conic given by $y^2 + 8x - 8y + 24 = 0$. Let E be the area of the triangle formed by the endpoints of the latus rectum and a point on the directrix.

Find $(A + B + C)D + E$.

Buchholz January Statewide 2018 Precalculus Team Round

Question 2

Consider the conic given by $9x^2 + 25y^2 - 54x + 100y = 44$. Let $Ax + By = C$ be the line with negative slope going through the point with maximal y-value and a focus, where A, B, and C are relatively prime and $A > 0$. Let D be the length of the latus rectum.

Consider another conic given by $y^2 + 8x - 8y + 24 = 0$. Let E be the area of the triangle formed by the endpoints of the latus rectum and a point on the directrix.

Find $(A + B + C)D + E$.

Buchholz January Statewide 2018 Precalculus Team Round

Question 3

Let $x - \log x - \ln x = 0$. $x \log_x 10$ can be expressed as $1 + \ln A$.

Let $\ln B$ be the value of $\ln|\sin\theta| + \ln|\cos\theta| + \ln|\cos 2\theta| + \ln|\cos 4\theta| + \ln|\cos 8\theta|$, evaluated at $\theta = \frac{3\pi}{64}$.

Find $A + 16B$.

Buchholz January Statewide 2018 Precalculus Team Round

Question 3

Let $x - \log x - \ln x = 0$. $x \log_x 10$ can be expressed as $1 + \ln A$.

Let $\ln B$ be the value of $\ln|\sin\theta| + \ln|\cos\theta| + \ln|\cos 2\theta| + \ln|\cos 4\theta| + \ln|\cos 8\theta|$, evaluated at $\theta = \frac{3\pi}{64}$.

Find $A + 16B$.

Question 4

Let $\sec\alpha = -\frac{13}{12}$, $\csc\alpha = \frac{13}{5}$, $\tan\beta = \frac{3}{4}$, and $\cos\beta = -\frac{4}{5}$.

A: $\sec(\alpha+\beta)$

B: $\sin(\beta-\alpha)$

C: $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}$

D: $\sin(2\beta)$

Find $ABCD$.

Buchholz January Statewide 2018 Precalculus Team Round

Question 4

Let $\sec\alpha = -\frac{13}{12}$, $\csc\alpha = \frac{13}{5}$, $\tan\beta = \frac{3}{4}$, and $\cos\beta = -\frac{4}{5}$.

A: $\sec(\alpha+\beta)$

B: $\sin(\beta-\alpha)$

C: $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}$

D: $\sin(2\beta)$

Find $ABCD$.

Buchholz January Statewide 2018 Precalculus Team Round

Question 5

Consider the following rational function: $f(x) = \frac{2x^3-3x^2-23x+12}{x^2-2x-8}$.

Let A be the number of vertical asymptotes of $f(x)$.

Let B be the number of horizontal asymptotes of $f(x)$.

Let $y = Cx + D$ be the equation of the slant asymptote.

Find $A + B - C + D$.

Buchholz January Statewide 2018 Precalculus Team Round

Question 5

Consider the following rational function: $f(x) = \frac{2x^3-3x^2-23x+12}{x^2-2x-8}$.

Let A be the number of vertical asymptotes of $f(x)$.

Let B be the number of horizontal asymptotes of $f(x)$.

Let $y = Cx + D$ be the equation of the slant asymptote.

Find $A + B - C + D$.

Question 6

Let A be the value of $(2+i)^2(1-i)^5(2-i)(1+i)^4$.

Let B be the sum of the non-real roots of $x^4+x^3-5x^2-7x+10$.

Find $A+B$.

Buchholz January Statewide 2018 Precalculus Team Round

Question 6

Let A be the value of $(2+i)^2(1-i)^5(2-i)(1+i)^4$.

Let B be the sum of the non-real roots of $x^4+x^3-5x^2-7x+10$.

Find $A+B$.

Buchholz January Statewide 2018 Precalculus Team Round

Question 7

Let A be the product of the solutions of $\cos(2x) + 5\sin x = 3$ for solutions in the interval $[0,2\pi]$.

Let B be the product of the solutions of $2\sec x = \tan x + \cot x$ for solutions in the interval $[0,2\pi]$.

Let C be the sum of the solutions of $\sin 2x = \sqrt{2}\cos x$ for solutions in the interval $[0,\pi]$.

Find $A - B + C$.

Buchholz January Statewide 2018 Precalculus Team Round

Question 7

Let A be the product of the solutions of $\cos(2x) + 5\sin x = 3$ for solutions in the interval $[0,2\pi]$.

Let B be the product of the solutions of $2\sec x = \tan x + \cot x$ for solutions in the interval $[0,2\pi]$.

Let C be the sum of the solutions of $\sin 2x = \sqrt{2}\cos x$ for solutions in the interval $[0,\pi]$.

Find $A - B + C$.

Question 8

Let A be $\sin\theta + \cfrac{1}{\cos\theta + \cfrac{1}{\sin\theta + \cfrac{1}{\cdots}}}$, where $\theta = \frac{\pi}{4}$.

Buffy's Ferris Wheel is 120 feet tall and has a radius of 50 feet. Alice, Helena, and Kira are currently in a car that is 70 feet above ground and is traveling at 25 feet per second. Let B be their angular velocity in radians per minute. (For simplicity, assume that the size of the car is negligible – the car can be represented by a point on the wheel.)

Find $\frac{A}{B}$.

Buchholz January Statewide 2018 Precalculus Team Round

Question 8

Let A be $\sin\theta + \cfrac{1}{\cos\theta + \cfrac{1}{\sin\theta + \cfrac{1}{\cdots}}}$, where $\theta = \frac{\pi}{4}$.

Buffy's Ferris Wheel is 120 feet tall and has a radius of 50 feet. Alice, Helena, and Kira are currently in a car that is 70 feet above ground and is traveling at 25 feet per second. Let B be their angular velocity in radians per minute. (For simplicity, assume that the size of the car is negligible – the car can be represented by a point on the wheel.)

Find $\frac{A}{B}$.

Question 9

Find a third degree polynomial in x with real coefficients such that two of its roots are 3 and $4+i$. Call this polynomial $f(x)$, where the leading coefficient is 1.

Let $\frac{3x^2+25x+23}{x^3+x^2-8x-12}=\frac{B}{x+2}+\frac{C}{(x+2)^2}+\frac{D}{x-3}$.

Find $f(B)+f(C)+f(D)$.

Buchholz January Statewide 2018 Precalculus Team Round

Question 9

Find a third degree polynomial in x with real coefficients such that two of its roots are 3 and $4+i$. Call this polynomial $f(x)$, where the leading coefficient is 1.

Let $\frac{3x^2+25x+23}{x^3+x^2-8x-12}=\frac{B}{x+2}+\frac{C}{(x+2)^2}+\frac{D}{x-3}$.

Find $f(B)+f(C)+f(D)$.

Question 10

Given the following system of equations, find $x + y + z$.

$$-3x + 4y = -3$$
$$2x - 2y + z = \frac{5}{2}$$
$$-2x - 5y - 4z = -\frac{3}{2}$$

Buchholz January Statewide 2018 Precalculus Team Round

Question 10

Given the following system of equations, find $x + y + z$.

$$-3x + 4y = -3$$
$$2x - 2y + z = \frac{5}{2}$$
$$-2x - 5y - 4z = -\frac{3}{2}$$

Question 11

For each triangle below, count how many distinct triangles can satisfy the given information. Then, add up the six numbers for your final answer.

Triangle ABC: $\overline{AB} = 5$, $\overline{BC} = 7$, $\angle B = 50°$

Triangle DEF: $\overline{DE} = 3$, $\overline{EF} = 7$, $\sin \angle E = 0.8$

Triangle GHI: $\overline{GH} = 5$, $\overline{HI} = 3$, $\sin \angle G = \frac{\sqrt{2}}{2}$

Triangle JKL: $\overline{JK} = 4$, $\overline{KL} = 9$, $\angle L = 10°$

Triangle MNP: $\overline{MN} = 5$, $\overline{NP} = 3$, $\sin \angle M = 0.2$

Triangle QRS: $\overline{QR} = 2$, $\overline{RS} = 3$, $\cos \angle R = 0.9$

Buchholz January Statewide 2018 Precalculus Team Round

Question 11

For each triangle below, count how many distinct triangles can satisfy the given information. Then, add up the six numbers for your final answer.

Triangle ABC: $\overline{AB} = 5$, $\overline{BC} = 7$, $\angle B = 50°$

Triangle DEF: $\overline{DE} = 3$, $\overline{EF} = 7$, $\sin \angle E = 0.8$

Triangle GHI: $\overline{GH} = 5$, $\overline{HI} = 3$, $\sin \angle G = \frac{\sqrt{2}}{2}$

Triangle JKL: $\overline{JK} = 4$, $\overline{KL} = 9$, $\angle L = 10°$

Triangle MNP: $\overline{MN} = 5$, $\overline{NP} = 3$, $\sin \angle M = 0.2$

Triangle QRS: $\overline{QR} = 2$, $\overline{RS} = 3$, $\cos \angle R = 0.9$

Question 12

Let A be the number of positive integer solutions to $a + b + c + d = 11$.

Let B be the probability that $\sin\theta \geq \cos 2\theta$, for $0 \leq \theta \leq 2\pi$.

Find AB.

Buchholz January Statewide 2018 Precalculus Team Round

Question 12

Let A be the number of positive integer solutions to $a + b + c + d = 11$.

Let B be the probability that $\sin\theta \geq \cos 2\theta$, for $0 \leq \theta \leq 2\pi$.

Find AB.

Buchholz January Statewide 2018 Precalculus Team Round

Question 13

A Gaussian integer is a complex number of the form $a + bi$, where a and b are both integers.

Let A be the number of Gaussian integers z with $|z| \leq 6$.

Let B be the number of real numbers x such that $\frac{30}{3+xi}$ is a Gaussian integer.

Find AB.

Buchholz January Statewide 2018 Precalculus Team Round

Question 13

A Gaussian integer is a complex number of the form $a + bi$, where a and b are both integers.

Let A be the number of Gaussian integers z with $|z| \leq 6$.

Let B be the number of real numbers x such that $\frac{30}{3+xi}$ is a Gaussian integer.

Find AB.

Buchholz January Statewide 2018 Precalculus Team Round

Question #14

Given that $\cos 36° = \frac{\sqrt{5}+1}{4}$,

A: $\sin 36°$

B: $\cos 36°$

C: $\sin 72°$

D: $\cos 72°$

Let E be the length of a non-side diagonal in a regular pentagon of side length 1.

Find $A^2 + B^2 + C^2 + D^2 + E$.

Buchholz January Statewide 2018 Precalculus Team Round

Question #14

Given that $\cos 36° = \frac{\sqrt{5}+1}{4}$,

A: $\sin 36°$

B: $\cos 36°$

C: $\sin 72°$

D: $\cos 72°$

Let E be the length of a non-side diagonal in a regular pentagon of side length 1.

Find $A^2 + B^2 + C^2 + D^2 + E$.