Answer key

1.	**C**	6.	**C**	11.	**C**	16.	**B**	21.	**E**	26.	**D**
2.	**B**	7.	**A**	12.	**C**	17.	**C**	22.	**B**	27.	**A**
3.	**B**	8.	**E**	13.	**D**	18.	**A**	23.	**E**	28.	**A**
4.	**D**	9.	**E**	14.	**A**	19.	**C**	24.	**B**	29.	**C**
5.	**D**	10.	**A**	15.	**C**	20.	**D**	25.	**A**	30.	**C**

Solutions

1. C—The value of limits agree when approaching from both sides, $\lim_{x\to 0^-} u(f(t)) = 0$, $\lim_{x\to 0^+} u(f(t)) = 0$.

2. B—$\pi \int_1^\infty \frac{1}{x^2} dx = \pi$. Half of it is $\frac{\pi}{2}$.

3. B—$\frac{((3)*(78+86)+(2)*(86+97)+(1)*(71))}{2} = 513$. Alternatively, the average of left-hand Riemann sum (503) and the right-hand Riemann sum (523) also arrives at 513.

4. D—Find the second derivative of the equation and set it to zero. We get $s'(t) = \frac{2t}{(1+t^2)^2}$, $s''(t) = \frac{2-6t^2}{(1+t^2)^3} = 0$ $t = \pm\frac{\sqrt{3}}{3}$.

5. D—$\frac{dy}{dx} = f'(x) = -3 = |\frac{\varepsilon}{\delta}|$ -> $\delta = \frac{\varepsilon}{3}$.

6. C— The particle reverses when the velocity changes sign. By setting the first derivative of s(t) =0 $6e^{-2t}\sin(2t) - 6e^{-2t}\cos(2t) = 0$, $\sin(2t) - \cos(2t) = 0$, $t = \frac{\pi}{8} + \frac{n\pi}{4}$ for $n \in \mathbf{Z}$.

7. A—First, substitute $x = -1$ into the $y^2 = -y + 0 \rightarrow y = -1, 0$. Then differentiate $2yy' = 3x^2y + x^3y' - \frac{\ln x^2}{2} - \frac{1}{2}x\frac{2x}{x^2}$. Finally, plug in point (-1, -1) and $(-1, 0)$ and get 4 and -1, respectively. The slope of the normal lines are $-\frac{1}{4}$ and 1.

8. E—By using the result of differentiation from the last problem, $2yy' = 3x^2y + x^3y' - \frac{\ln x^2}{2} - 1$, we continue the evaluation by isolating $y' \rightarrow y' = \frac{3x^2y - \frac{\ln x^2}{2} - 1}{2y - x^3}$. To have a vertical tangent, $2y - x^3 = 0$. Now trace back to the original equation $y^2 = x^3y - \frac{x\ln x^2}{2}$. Substitute $y = \frac{x^3}{2} \rightarrow \frac{x^6}{4} = \frac{x^6}{2} - \frac{x\ln x^2}{2}$. Quickly see that x is a factor. Giving 0 as a solution. However, 0 is not within the domain due to $\ln x^2$.

9. E— The point of our interest is in the first quadrant, so we can quickly find that to be $(1,1)$. First find y' by plugging the point in $2yy' = 3x^2y + x^3y' - \frac{\ln x^2}{2} - 1 \rightarrow\ y' = 2$. Then differentiate again, $2(y')^2 + 2yy'' = 6xy + 3x^2y' + 3x^2y' + x^3y'' - \frac{1}{x} \rightarrow 8 + 2y'' = 6 + 6 + 6 + y'' - 1 \rightarrow y'' = 9$

10. A—Because $x, y\ \in\ (0,1)$, we can find the bound for $y\ =\ 1 = 3x^2$ -> $x\ = \frac{1}{\sqrt{3}}$ Integrate the geometric probability graph $\int_0^{\frac{1}{\sqrt{3}}}(1 -\ 3x^2)dx\ = \frac{2\sqrt{3}}{9}$.

11. C—$V\ =\ \frac{4}{3}\ \pi r^3\quad dv = 4\pi r^2 dr\ =\ 4\pi * 3 * 3 * 1 = 36\pi$.

12. C—Rolle's Theorem states that $f'(c)\ =\ 0$. Since $f(x)\ =\ cos(x)$, $f'(x) = -sin(x)$. By setting $f'(x) =\ 0$, obtain c = $n\pi$ for n $\in$ **Z** So, 2π is the only correct answer.

13. D—Rewrite and take the second derivative of $f(x)$, $f''(x) = 30x^4 - 360x^2 + 480x = 30x(x+4)(x-2)^2$ A point of inflection exists when there is a change in sign on the second derivative of a function. Because at $x = 0, -4, f''(x)$ a sign change occurs, there are $f(x)$ has 2 points of inflections.

14. A—Let $y = \lim_{x\to 1} x^{\left(\frac{2}{ln(x^2)}\right)}\qquad ln(y) = \lim_{x\to 1}\left(\frac{2\ ln(x)}{ln(x^2)}\right) =\ \lim_{x\to 1}\left(\frac{2\ ln(x)}{2\ ln(x)}\right) = 1\quad y =\ e^1 = e.$

15. C—$\lim_{x\to 1}\frac{1}{x-1} - \frac{1}{ln(x)} = \frac{ln(x)-x+1}{(x-1)ln(x)}$. Directly plug in $x = 1$ gives $\frac{0-1+1}{(1-1)1} = \frac{0}{0}$, the conditions to apply L'Hôpital's rule is satisfied. $\frac{\frac{1}{x}-1}{ln(x)+x\frac{1}{x}-\frac{1}{x}}$ Again, $\frac{-\frac{1}{x^2}}{\frac{1}{x}+\frac{1}{x^2}} = -\frac{1}{2}$

16. B—Chain Rule: $\frac{d}{dx}\big(f(x^3)\big) = 3x^2 g(x^3)$ Product Rule: $\frac{d^2}{dx^2} = \frac{d}{dx}\left(\frac{d}{dx}\big(f(x^3)\big)\right) = \frac{d}{dx}\big(3x^2 g(x^3)\big) = 9x^4 f(x^6) + 6xg(x^3)$.

17. C—$f'(x) = 4x^3 - 2x + C_1\quad f'(-1) = -4\quad C_1 = -2\quad f(x) = x^4 - x^2 - 2x + C_2\quad f(0) = 2$ Then plug in the initial condition $f(2) = 16 - 4 - 4 + 2 = 10$.

18. A—$f(x) =\ (\sin(x))^{f(x)}\quad f'(x) = (f(x)\cot(x) + f'(x)\ln(\sin(x)))\cdot f(x)$. At $x\ = \frac{\pi}{2}, y = 1$. Plugging in the values yields $f'\left(\frac{\pi}{2}\right) = 0$

19. C—The first derivative simplifies as follows:

$$\frac{d(g(x))}{d(f(x))} = \frac{\frac{d(g(x))}{dx}}{\frac{d(f(x))}{dx}} = \frac{sec^2(x)+\sec(x)\tan(x)}{\sec(x)} = g(x).$$

Thus all higher order derivatives will produce the same result, and the answer is $g(x)$

20. D— $\int_0^2 f(x)dx =\ \int_0^3 f(x)dx +\ \int_3^6 f(x)dx -\ \int_2^6 f(x)dx =\ 3 - 6 + 2 =\ -1$

21. E—The definition of e can be manipulated $\lim_{x\to-\infty}(1+\frac{a}{x})^{bx} = e^{abx}$. In this case, $a = -\frac{1}{2}$, $b = 6$. So the answer will be e^{-3}.

22. B—We can call a variable point on the parabola (a, a^2). Then, the distance between $(0, 4)$ to $(a, \frac{a^2}{2})$ is $d = \sqrt{(0-a)^2 + (\frac{a^2}{2} - 4)^2}$. To minimize d^2, set the first derivative to zero. $a = 0, -\sqrt{6}, \sqrt{6}$. Lastly, do not forget to plug in a and get $d = \sqrt{7}$.

23. E—The second fundamental theorem of calculus holds for the question. $F'(0) = 3e^{81*0^4} = 3$.

24. B—The integral evaluates to $-\cos(sin(x))$ from 0 to $\frac{\pi}{2}$ is equal to $1 - cos(1)$.

25. A—$average\ acceleration = \frac{v(2)-v(1)}{1} = 3*2^2 - 3*1^2 = 9$.

26. D—The shape of the solid is half of an ellipsoid and its volume is given by the formula $V = \frac{4}{3}abc\pi$ while a, b, c is half of the length of each axis, respectively. $V = \frac{1}{2}\frac{4}{3} * 3 * 4 * 4\pi = 32\pi$.

27. A—Rewrite the differential equation and get $\frac{dy}{dx} = y$. By separation of variables, $\int\frac{dy}{y} = \int dx$. $\ln|y| = x + C$. Plug in the initial condition $y(0) = 1$ and find $C = 0$. Therefore, $y(1) = e$.

28. A—By the Chain Rule, evaluate the indefinite integral $-\frac{1}{2}e^{-x^2}$ from 0 to ∞. The result is $\frac{1}{2}$.

29. C—It is well known that the Simpson's Rule is derived from quadratic function. It not only computes the exact value of a second-degree function but also calculates cubic functions perfectly. $\int_0^2 x^3 - 3x^2 + 4x\,dx = 4$.

30. C—Drawing derivatives implicitly often results in clean answers. $3x^2 + 3y^2y' = 0$. $y' = -\frac{x^2}{y^2}$.

$6x + 6y(y')^2 + 3y^2y'' = 0$. Substitute $y' = -\frac{x^2}{y^2}$ into $6x + 6y(y')^2 + 3y^2y'' = 0$ and simplify

$y'' = -\frac{2x(x^3+y^3)}{-y^5} = -\frac{2x}{y^5}$