

Helen Dostal Test
2017

The abbreviation "NOTA" means
"None of These Answers."

NO CALCULATOR is allowed on this test.

Scoring is 4 points for a correct answer,
0 points for an a problem left unanswered
(blank), and -1 for a wrong answer.

- For a, b and c real numbers,
 $(ax+3)(bx+2) = 15x^2 + cx + 6$ and
 $a+b = 16$, find the two possible
values of c .
A. 1 or 15
B. 5 or 75
C. 21 or 33
D. 33 or 47
E. NOTA
- A parabola has x -intercepts 3 and -2
and vertex $\left(\frac{1}{2}, 6\right)$. If the equation of the
curve is $y = ax^2 + bx + c$, give the value
of $a+b+c$.
A. $\frac{24}{25}$ B. $\frac{144}{25}$
C. $\frac{192}{125}$ D. $\frac{48}{25}$
E. NOTA
- Thirty-one more than the square root of
a number is 25 more the number. Which
of the following is the greatest prime
factor of the number?
A. 3 B. 5
C. 7 D. 13
E. NOTA
- For $4\sqrt{45} + \sqrt{80} - \sqrt{x} = \sqrt{500}$, give the
sum of the digits of x .
A. 5 B. 6
C. 9 D. 11
E. NOTA
- For $i = \sqrt{-1}$, find the value of $\frac{(2-3i)^2}{1+i}$
is equal to $a+bi$ for rational values a
and b . Give the value of $a+b$.
A. -21 B. -15
C. -12 D. -9
E. NOTA
- For the inequality $2(x^2 + 6) \leq 11x$
solved over the whole numbers,
give the sum of the least and the
greatest numbers in the solution set.
A. 5 B. 6
C. 7 D. 8
E. NOTA
- The function $f(x) = 2x^2 - 3x + k$ has two
distinct non-real roots. Which can NOT
be a value for k ?
A. $\frac{17}{8}$ B. 2
C. $\frac{100}{7}$ D. 12
E. NOTA
- Which of the following coordinates
determine a point where a focus of the
graph of $25x^2 + 9y^2 - 100x - 18y - 116 = 0$
lies?
A. (6, 1) B. (2, -3)
C. (2, 4) D. (2, 1)
E. NOTA

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9. Consider a 12-hour digital clock, and all of the times which are palindromic when written digitally, such as 1:41 or 12:21. The times can be three- or four-digit. What is the smallest time interval between two times if both of the times are palindromic?
- A. 1 hour, 1 minute
B. 10 minutes
C. 1 hour, 10 minutes
D. 1 hour
E. NOTA

10. The system below has solution (X, Y, Z) .

$$\begin{cases} 5x - y + 2z = 37 \\ 3x + y - 4z = -23 \\ x - 2y + 5z = 47 \end{cases}$$

Give the value of $Z - Y - X$.

- A. 4 B. 5
C. 6 D. 7
E. NOTA
11. Given $5\sin^2 x + 3 = 8\sin x$, find the sum of all possible real values of $\tan x$, where defined, for $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$.

- A. $-\frac{3}{4}$ B. 0
C. $\frac{\pi}{2} - \frac{3}{4}$ D. $\frac{3}{4}$
E. NOTA

12. Ten students stand in a room. Six of them are male and four of them are female. If three students are chosen at random, what is the probability that two are male and one female?

- A. $\frac{1}{5}$ B. $\frac{1}{4}$
C. $\frac{1}{3}$ D. $\frac{1}{2}$
E. NOTA

13. Give the product of all solutions to the equation when it is solved over Reals:

$$\sqrt{9x^2 + 6x + 1} + x = 7$$

- A. -1.5 B. -4
C. -6 D. 1.5
E. NOTA

- 14.



Bernice the lamb is attached by a 9 ft leash to vertex of the hexagonal floor of a silo. Each side of the hexagon is 6 feet long. Ignoring Bernice's dimensions, find out the area in square feet of the ground that she can roam outside of the silo.

- A. 27π B. 52π
C. 54π D. 57π
E. NOTA

15. Consider all whole numbers less than 100 as the Universal Set U . Subset L consists of all elements in U which are 1 greater than a multiple of 4. Subset V consists of all elements in U which are 1 less than a multiple of five. Give the sum of all elements in set $L \cap V$ which are **prime**.

A. 187 B. 185
C. 121 D. 118
E. NOTA

16. $(x+2)^3 - (x+2)^2 - 2(x+2) + 2 = 0$ has three real solutions, x_1, x_2 , and x_3 , for $x_1 < x_2 < x_3$. Find the value of x_3 .

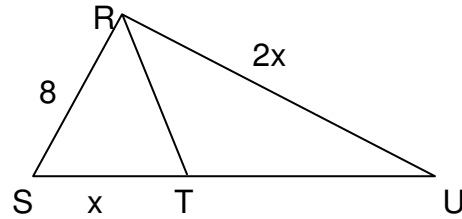
A. -1 B. $-\sqrt{2}$
C. $\sqrt{2}-2$ D. $-\sqrt{2}-2$
E. NOTA

17. Which is an expression equal to

$$\frac{4^{x+1} - 2(2)^{2x-1}}{2^{2x+3}} \text{ for all real values of } x.$$

A. $\frac{3}{8}$ B. $\frac{2^x}{3}$
C. $\frac{5}{8}$ D. $\frac{2}{3}$
E. NOTA

18.



\overline{RT} bisects $\angle SRU$, and T is between S and U . $SR = 8$, $RU = 2x$, $ST = x$ and $SU = 15$. Give the perimeter of $\triangle SUR$.

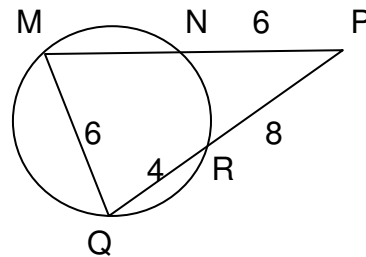
A. 27 B. 29
C. 33 D. 35
E. NOTA

19. Give the sum of all distinct values of θ for $0 < \theta \leq 2\pi$ that makes the equation

$$(2\sin^2(\theta) - 1)^{2\sin^2(\theta) - 3\sin(\theta) + 1} = 1 \text{ true.}$$

A. 3π B. 3.16π
C. 3.5π D. 4π
E. NOTA

20.



Secant segments \overline{MP} and \overline{QP} intersect the circle shown at points N and R respectively. $NP=6$, $PR=8$, $QR=4$, and $MQ=6$. What is the perimeter of $\triangle PMQ$?

A. 34 B. 32
C. 30 D. 27
E. NOTA

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21. $\triangle RUG$ has side lengths 6, 8 and 12. What is an expression for the measure of the largest angle of $\triangle RUG$. (The traditional restricted range for $y = \text{Arc cos}(x)$ is used.

A. $\pi - \text{Cos}^{-1}\left(\frac{43}{48}\right)$ B. $\pi - \text{Cos}^{-1}\left(\frac{11}{24}\right)$
C. $\text{Cos}^{-1}\left(\frac{11}{24}\right)$ D. $\text{Cos}^{-1}\left(\frac{43}{48}\right)$
E. NOTA

22. For $\pi < \alpha < \frac{3\pi}{2}$ and $\sin \alpha = -\frac{4}{5}$, and $\cos \beta = \frac{1}{2}$ for $0 < \beta < \frac{\pi}{2}$, give the value of $\sin(\alpha + 2\beta)$.

A. $\frac{-3-4\sqrt{3}}{10}$ B. $\frac{3-4\sqrt{3}}{10}$
C. $\frac{-4-3\sqrt{3}}{10}$ D. $\frac{4-3\sqrt{3}}{10}$
E. NOTA

23. Give the constant term of the

expansion of $\left(\frac{2}{x^2} - \frac{x^4}{2}\right)^9$.

A. -672 B. -84
C. 168 D. 336
E. NOTA

24. A shirt is on sale for 10% off. Ralph bought the shirt, paying the discounted price, plus 6% sales tax on the discounted price. If Ralph paid p for the total, then what was the original price of the shirt, before discount?

A. $\frac{1537p}{1250}$ B. $\frac{29p}{25}$
C. $\frac{25p}{16}$ D. $\frac{500p}{477}$
E. NOTA

25. Given that $4x - y = -3$, find the value

of $\frac{2^y}{16^x}$.

A. $-\frac{1}{4}$ B. $\frac{1}{8}$
C. 8 D. 24
E. NOTA

26. If the expression $\frac{\frac{b}{a^2-b^2}}{\frac{1}{a+b} - \frac{1}{a-b}}$ is

simplified for $|a| \neq |b| \neq 0$, then give the simplified expression.

A. $\frac{1}{a}$ B. $-\frac{1}{2}$
C. -1 D. b
E. NOTA

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27. A pool has two drains (D_1 and D_2) as well as two faucets (F_1 and F_2). When the pool is empty, faucet F_1 can fill it completely in 4 hours alone, and F_2 can fill it completely alone in 5 hours. When the pool is full, drain D_1 alone can empty it in 10 hours, and D_2 can empty it in 12 hours. At noon today, the pool was empty and F_1 was turned on. After an hour, F_2 was also turned on, and after another hour, both drains were accidentally opened. What is the time at which the least elapsed time occurs when the pool is full?
- A. 3:07:30 PM B. 1:07:30 PM
C. 12:33:16 PM D. 12:09:08 PM
E. NOTA

28. A three-digit positive number has middle digit 1. When I reverse the digits, I get a number that is 198 less than the original three-digit number. If the digits of the original number are P, 1 and Q, and the original number is written P1Q, then which statement must be true, given this information?
- A. P or Q must be even.
B. P or Q must be odd.
C. $|P-Q|$ must be 2.
D. $|P-Q|$ must be 1.
E. NOTA

29. An isosceles triangle has perimeter 48 and one side has length 18. Give the least possible area of the triangle.
- A. 108 B. $72\sqrt{2}$
C. $48\sqrt{3}$ D. 54
E. NOTA
30. If $(3x^2 - 4x + 2)(5x - 1)^2$ is expanded and put in the form $ax^4 + bx^3 + cx^2 + dx + k$ then give the value of $a + 2b + 3c + 4d + 5k$.
- A. 8 B. 96
C. 100 D. 110
E. NOTA