

- 1) Find the maximum value of the function $f(x)=2x^2e^x$ on the interval $(-\infty,0]$.
- 2) Suppose that the variable y varies inversely as x^2 , and suppose that x is decreasing at a constant rate of 5 units/sec. At what rate (in units/sec.) does y change when x and y are equal?
- 3) The equation $xy+xy^2=2x^2+18$ defines y implicitly as a function of x . Find y'' at the point $(3,3)$.
- 4) Evaluate the integral $\int_0^1 \frac{1+x}{1+x^2} dx$.
- 5) Find the slope of the tangent line to the curve defined by the equation $x^2 + y^2 - 8x + 6y + 20 = 0$ at the point $(3, -1)$.
- 6) It is known that $\int_0^1 y^\alpha (1-y)^\beta dy = \frac{\alpha! \beta!}{(\alpha + \beta + 1)!}$, when α, β are positive integers. Use this fact to evaluate:
$$\int_0^1 y^2 (1 - 2y + y^2) [(1 - y)^{18} + 10y^2] dy$$
.
- 7) Farmer John must make a silo in the shape of a cylindrical tube topped by a hemispherical dome of equal radius. If the structure must be able to store his crop of 9π cubic meters of grain, what radius (in meters) will give his silo the smallest surface area.
- 8) A rectangle has perimeter 32 and diagonal 12. Find its area.
- 9) For all x , it is true that $(x-1)(1+x+x^2+\dots+x^n) = x^{n+1} - 1$.
If the sum $1 + 2 \cdot 2 + 3 \cdot 2^2 + 4 \cdot 2^3 + 5 \cdot 2^4 + \dots + 31 \cdot 2^{30}$ is expressed in the form $A \cdot 2^B + C$, then what is $A - B + C$?
- 10) The average value of $f(x)$ on the interval $[2,5]$ is $\frac{4}{3}$. What is the area between the curves $y=x$ and $y=f(x)$ on this interval if $x \geq f(x)$ for all x ?

11) Find the arithmetic mean of all values of k such that

$$\int_0^k (25 - 70x + 60x^2 - 16x^3) dx = 6.$$

12) On what interval(s) from $[0, 2\pi)$ is the graph of $y = x + \sin x + \sqrt{3} \cos x$ decreasing?

13) Find $\int (\ln x)^2 dx$.

14) Evaluate $\int_{\pi/4}^{\pi/2} (\sin^6 x - \sin^2 x \cos^4 x + \sin^4 x \cos^2 x - \cos^6 x) dx$.

15) Find $\lim_{n \rightarrow \infty} (n - \sqrt{n^2 - 2n})$.

Calculus Team Answers

Cypress 97 Invitational

1) $8/e^2$

2) 10

3) $4/21$

4) $\pi/4 + (1/2) \ln 2$

5) $1/2$

6) $169/1771$

7) $(9/2)^{1/3}$

8) 56

9) 0

10) $13/2$

11) $5/4$

12) $(\pi/3, \pi)$

13) $x(\ln x)^2 + 2x(\ln x) + 2x + C$

14) $1/2$

15) 1