

1997 Coral Springs Invitational Precalculus Answers

Individual Test Answers

- | | |
|--------|--------|
| 1. D. | 16. B. |
| 2. A. | 17. C. |
| 3. C. | 18. B. |
| 4. B. | 19. D. |
| 5. A. | 20. E. |
| 6. D. | 21. C. |
| 7. B. | 22. C. |
| 8. B. | 23. E. |
| 9. D. | 24. B. |
| 10. B. | 25. C. |
| 11. E. | 26. D. |
| 12. C. | 27. B. |
| 13. D. | 28. C. |
| 14. A. | 29. E. |
| 15. A. | 30. B. |

Team Round Answers

- | | |
|-----|--------------|
| 1. | 1/4 |
| 2. | 2 |
| 3. | 8.41 |
| 4. | DCBA |
| 5. | 10 |
| 6. | 90 |
| 7. | 4 |
| 8. | 16384 |
| 9. | 31.5 |
| 10. | 80.5 minutes |
| 11. | 76 |
| 12. | 248 mph |
| 13. | 6π |
| 14. | $(0, 2\pi)$ |
| 15. | -11 |

Answer Breakdown

4 A's

9 B's

7 C's

6 D's

4 E's

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1. $1/4$

$$25x^2 + 16y^2 - 100x + 32y - 284 = 0$$

$$\frac{(x-2)^2}{16} + \frac{(y+1)^2}{25} = 1$$

$$A = c/a = 3/5$$

$$B = 2b^2/a = 32/5$$

$$C = c = 3$$

$$D = 2b = 8$$

$$\frac{\left(\frac{3}{5}\right)(8)}{\left(\frac{32}{5}\right)(3)} = \frac{1}{4}$$

2. 2

$$\begin{vmatrix} 4 & 6 & -6 & 9 \\ 1 & M & 5 & -3 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{vmatrix} = -2$$

$$\begin{vmatrix} 4 & 6 & 9 \\ 1 & M & -3 \\ 0 & 0 & 1 \end{vmatrix} = 2$$

$$\begin{vmatrix} 4 & 6 \\ 1 & M \end{vmatrix} = 2$$

$$4M - 6 = 2$$

$$M = 2$$

3. 8.41

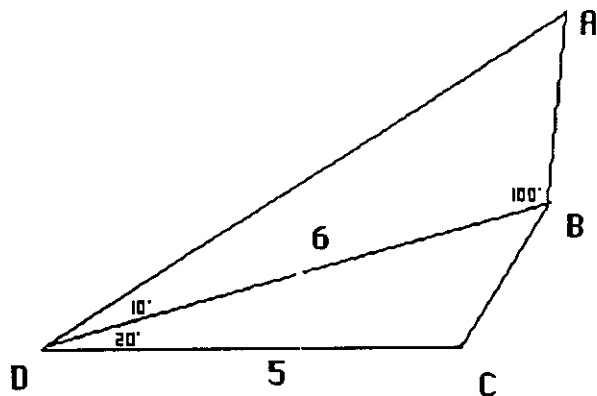
$$\text{Area } \triangle BCD = .5(6)(5)(\sin 20^\circ) \approx 5.1303$$

$$AD/\sin 100^\circ = 6/\sin 70^\circ$$

$$AD = 6.28\dots$$

$$\text{Area } \triangle DBA = .5(6)(6.28\dots)(\sin 10^\circ) \approx 3.2757$$

$$\text{Area of } ABCD = 8.41$$



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4. DCBA

$$A = 10111011_2 = 187$$

$$B = 215320_7 = 37891$$

$$C = 35554_{11} = 51242$$

$$D = 6026_9 = 4398$$

$$A+B+C+D = 93718_{10}$$

$$93718_{10} = DCBA_{19}$$

5. 10

$$\sqrt{2+\sqrt{3}} = \frac{\sqrt{a}+\sqrt{b}}{c} = \sqrt{\frac{a}{c^2}} + \sqrt{\frac{b}{c^2}}$$

$$2+\sqrt{3} = \frac{a}{c^2} + \frac{b}{c^2} + 2\sqrt{\frac{ab}{c^4}}$$

$$\left(\frac{a}{c^2}\right) + \left(\frac{b}{c^2}\right) = 2$$

$$\text{Let } A = \frac{a}{c^2}, \text{ let } B = \frac{b}{c^2}$$

$$A+B=2$$

$$4AB=3$$

$$A, B = \frac{3}{2}, \frac{1}{2}$$

$$\frac{\sqrt{a}+\sqrt{b}}{c} = \sqrt{\frac{a}{c^2}} + \sqrt{\frac{b}{c^2}} = \sqrt{A} + \sqrt{B} = \sqrt{\frac{3}{2}} + \sqrt{\frac{1}{2}} = \frac{\sqrt{3}+\sqrt{1}}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{2}$$

6. 90

$$A = (6-1)! / 2 = 60$$

$$B = 9C_5 - 6C_5 = 120$$

$$C = (8-1)! = 5040$$

$$D = (7*8) / 2 = 28$$

$$(60)(5040) / (120)(28) = 90$$

7. 4

Let roots equal a,b,c,d,e,f

$$a^2+b^2+c^2+d^2+e^2+f^2 = (a+b+c+d+e+f)^2 - 2(ab+ac+bc\dots)$$

$$= (0)^2 - 2(-2) = 4$$

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8. 16384

$$A = \frac{15}{2}(14 + (14)(6)) = 735$$

$$B = \frac{\frac{1}{3}}{1 - \frac{1}{3}} = \frac{1}{2}$$

$$C = 2^{(1+2+3+4+5)} = 32768$$

$$D = \frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} \dots \times \frac{1}{\infty} = 0$$

$$A + B + C + D = 16384$$

9. 31.5

Break hexagon into 4 triangles.

$$\frac{1}{2} \begin{vmatrix} -1 & 3 & 1 \\ 1 & 8 & 1 \\ -3 & 7 & 1 \end{vmatrix} + \frac{1}{2} \begin{vmatrix} -1 & 3 & 1 \\ 4 & 6 & 1 \\ 1 & 8 & 1 \end{vmatrix} + \frac{1}{2} \begin{vmatrix} -1 & 3 & 1 \\ 3 & 2 & 1 \\ 4 & 6 & 1 \end{vmatrix} + \frac{1}{2} \begin{vmatrix} -1 & 3 & 1 \\ -2 & 1 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 31.5$$

10. 80.5 minutes

$$a^2 = 36^2 + 13.5^2 - 2(36)(13.5)(\cos 23^\circ)$$

$$a = 24.15614\dots$$

$$t = (24.15614\dots) / (18) = 1.342 \text{ hours}$$

$$t = 1.342 * 60 = 80.52\dots \text{ minutes}$$

11. 76

Let sides equal a,b,c.

$$SA = 2ab + 2bc + 2ac = 236$$

$$\text{diagonal}^2 = a^2 + b^2 + c^2 = 125$$

$$\text{sum of lengths} = 4a + 4b + 4c$$

$$a^2 + b^2 + c^2 + 2ab + 2bc + 2ac = (a+b+c)^2 = 361$$

$$a+b+c = 19$$

$$4a+4b+4c = 76$$

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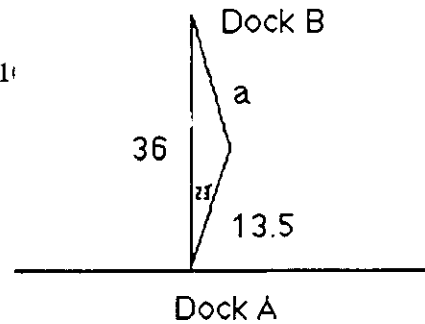
12. 248 mph

vector of plane = $\langle 0, 400 \rangle$

vector of wind = $\langle 200 \cos 300^\circ, 200 \sin 300^\circ \rangle = \langle 100, -173.2 \rangle$

actual flight path vector = $\langle 0 + 100, 400 - 173.2 \rangle$

airspeed = magnitude = $247.8627... = 248$ mph



13. 6π

$$(\sin x)(\sin 3x) + \cos 4x = 0$$

$$(\sin x)(3\sin x - 4\sin^3 x) + (8\sin^4 x - 8\sin^2 x + 1) = 0$$

$$4\sin^4 x - 5\sin^2 x + 1 = 0$$

$$(4\sin^2 x - 1)(\sin^2 x - 1) = 0$$

$$\sin x = \pm 1/2, \sin x = \pm 1$$

$$x = \pi/6, 5\pi/6, 7\pi/6, 11\pi/6, \pi/2, 3\pi/2$$

$$\text{Sum} = 6\pi$$

14. $(0, 2\pi)$

$$1 = \cos^2 \theta + 0.5(1 - \cos 3\theta \cos \theta - \sin 3\theta \sin \theta)$$

$$= \cos^2 \theta + 0.5(1 - \cos 2\theta)$$

$$= \cos^2 \theta + \sin^2 \theta$$

all real values are true

15. -11

If G and F are inverses then $G(F(x)) = x$.

Therefore $G(F(F(x))) = F(x)$.

$G(F(F(1)))$ then equals $F(1) = -11$.