CALCULUS SOLUTIONS

B 1. $A = (1+t)\pi$ and for a circle, $A = \pi r^2$ so that $r = \sqrt{1+t}$
$\Rightarrow \frac{dr}{dt} = \frac{1}{2\sqrt{1+t}}$.
The circle is tangent to the x-axis when $t = 15$, and at this instant, $\frac{dr}{dt} = \frac{1}{2\sqrt{1+15}} = \frac{1}{8}$

D 2. $2\int_{\pi/3}^{\pi} \cos\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right)\frac{1}{2}\,dx = 2\cdot\frac{1}{2}\sin^2\left(\frac{x}{2}\right)\Big]_{\pi/3}^{\pi} = \sin^2\left(\frac{\pi}{2}\right) - \sin^2\left(\frac{\pi}{6}\right) = \frac{3}{4}$

A 3 $x^2 + x - 12 > 0 \Rightarrow (x+4)(x-3) > 0$ which means
$x < -4$ or $x > 3$: domain is $(-\infty, -4)\cup(3,\infty)$

A 4. Graphs intersect when $x^2 = \frac{2}{1+x^2}$. Cross-multiplying, we get
$x^4 + x^2 - 2 = 0$
$(x^2+2)(x^2-1) = 0$
$x = \pm 1$
so the graphs intersect at $(-1,1)$ and $(1,1)$.
Also, when $-1 < x < 1$, $\frac{2}{1+x^2} > x^2$.
area $= \int_{-1}^{1}\left(\frac{2}{1+x^2} - x^2\right)dx = 2\,\text{Arctan}\,x - \frac{1}{3}x^3\Big]_{-1}^{1} = 2\cdot\frac{\pi}{4} - \frac{1}{3} - \left(2\cdot\frac{-\pi}{4} + \frac{1}{3}\right) = \pi - \frac{2}{3}$

C 5. This limit equals the derivative of $f(g(x))$ at $x = 1$, which is $f'(g(1))\,g'(1) = (-2)(1) = -2$

B 6. $3e\cos\pi$ is a constant, so $\frac{dy}{dx} = 0$

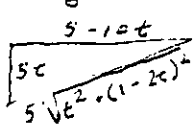
B 7. $y' = 2\tan x\sec^2 x$, so the slope of the tangent line is
$2\tan\frac{\pi}{4}\sec^2\frac{\pi}{4} = 2(1)(\sqrt{2})^2 = 4$

B 8. $v = s' = 6t^2 - 24t$ and $a = v' = 12t - 24$.
$a = 0$ when $12t - 24 = 0 \Rightarrow t = 2$

D 9. $f'(x) = \frac{1/2\sqrt{x}}{\sqrt{1-x}} = \frac{1}{2\sqrt{x-x^2}}$ so $f'\left(\frac{1}{5}\right) = \frac{1}{2\sqrt{\frac{1}{5}-\frac{1}{25}}} = \frac{1}{2\sqrt{\frac{4}{25}}} = \frac{1}{2\cdot\frac{2}{5}} = \frac{5}{4}$

A 10. $\int_{\pi/6}^{\pi/3}\cot x\,dx = \ln|\sin x|\Big]_{\pi/6}^{\pi/3} = \ln\left(\frac{\sqrt{3}}{2}\right) - \ln\left(\frac{1}{2}\right) = \ln\sqrt{3}$

B 11. Average value $= \frac{1}{6}\int_1^7 x(x^2+1)^{-1/2}dx = \frac{1}{6}\cdot\frac{1}{2}\cdot 2(x^2+1)^{1/2}\Big]_1^7 = \frac{1}{6}(50^{1/2} - 2^{1/2})$
$= \frac{1}{6}(5\sqrt{2} - \sqrt{2}) = \frac{2\sqrt{2}}{3}$

B 12.


We need to minimize $t^2 + (1-2t)^2$:
$f(t) = t^2 + (1-2t)^2 = t^2 + 1 - 4t + 4t^2 = 5t^2 - 4t + 1$
$f'(t) = 0 \Rightarrow 10t - 4 = 0 \Rightarrow t = 2/5$
$2/5$ of an hour past noon is 12:24

A 13. $P_2 = \frac{1}{2^2 2!}\frac{d^2}{dx^2}(x^2-1)^2 = \frac{1}{8}\left(\frac{d^2}{dx^2}\left(x^4 - 2x^2 + 1\right)\right) = \frac{1}{8}(12x^2 - 4) = \frac{3}{2}x^2 - \frac{1}{2}$
$P_2\left(\frac{1}{2}\right) = \frac{3}{2}\left(\frac{1}{4}\right) - \frac{1}{2} = -\frac{1}{8}$

C 14. $f'(x) = 6x^2 + 2x - 20 = 2(3x-5)(x+2)$ $\begin{array}{ccccc} - & & + & & - \\ \hline & -2 & & 5/3 & \end{array}$ so minimum at $x = \frac{5}{3}$

C 15 $\ln y = \sqrt{x}\ln x$ so $\frac{y'}{y} = \frac{\sqrt{x}}{x} + \frac{\ln x}{2\sqrt{x}}$. At $x = 4$, $y' = 4^{\sqrt{4}}\left(\frac{1}{2} + \frac{\ln 4}{4}\right) =$
$16\left(\frac{1}{2} + \frac{1}{2}\ln 2\right) = 8 + 8\ln 2$

C 16. $\int_0^{\pi/3} \tan^3 x(1+\tan^2 x)\sec^2 x\,dx = \int_0^{\pi/3} (\tan^3 x\sec^2 x + \tan^5 x\sec^2 x)\,dx =$
$\frac{1}{4}\tan^4 x + \frac{1}{6}\tan^6 x\Big]_0^{\pi/3} = \frac{9}{4} + \frac{27}{6} = \frac{9+18}{4} = \frac{27}{4}$

E 17. $\frac{x+4}{(x-2)(x+1)} = \frac{2}{x-2} - \frac{1}{x+1}$ so $\int_0^1 \left(\frac{2}{x-2} - \frac{1}{x+1}\right)dx = 2\ln|x-2| - \ln|x+1|\Big]_0^1 =$
$2\ln 1 - \ln 2 - 2\ln 2 + \ln 1 = -3\ln 2$

D 18. $\sec(x - \pi/4)$ is undefined when $\cos(x-\pi/4) = 0$. $\cos x = 0$ when $x = \frac{\pi}{2}$ so $\frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$.

A 19. $2\pi\int_0^1 (2-x)(x-x^3)\,dx = 2\pi\int_0^1 (2x - x^2 - 2x^3 + x^4)\,dx =$
$2\pi\left(x^2 - \frac{1}{3}x^3 - \frac{1}{2}x^4 + \frac{1}{5}x^5\right)\Big]_0^1 = 2\pi\left(1 - \frac{1}{3} - \frac{1}{2} + \frac{1}{5}\right) = \frac{2\pi}{30}(30 - 10 - 15 + 6) = \frac{11\pi}{15}$

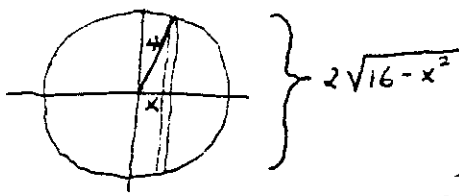
C 20. $f'(x) = \frac{2x}{\sqrt{4x^2+9}}$ so $\lim_{x\to\infty} f'(x) = 1$

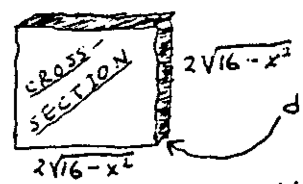
C 21. $y' = 3\cos x - 4\sin x$ which means y has a maximum when $3\cos x = 4\sin x$ in the first quadrant: $\tan x = \frac{3}{4} \Rightarrow x = \text{Arctan}\,\frac{3}{4}$.

D 22. $\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0 \Rightarrow x^{-1/3} + y^{-1/3}y' = 0 \Rightarrow y' = -\frac{x^{-1/3}}{y^{-1/3}} = -\frac{y^{1/3}}{x^{1/3}}$.
At $(-1, 8)$, $y' = -\frac{2}{-1} = 2$

A 23. $f(-2) = 4 - 2 + 6 = 8 = -4 - 2a + b$
$f'(-2) = 2(-2) + 1 = -3 = -2(-2) + a$
so $\begin{cases} -2a + b = 12 \\ a = -7 \end{cases}$ $\therefore b = -2$
$a - b = -7 + 2 = -5$

D 24. Give circle a name: $x^2 + y^2 = 16$.





The volume of each sliver is $V = (2\sqrt{16-x^2})^2\,dx$

total volume $= \int_{-4}^{4} (2\sqrt{16-x^2})^2\,dx = 2\int_0^4 (4)(16-x^2)\,dx =$
$8\left[16x - \frac{1}{3}x^3\right]_0^4 = 8\left(64 - \frac{64}{3}\right) = \frac{1024}{3}$

A 25. $\frac{dx}{dt} = \frac{3}{2}t^{1/2}$ and $\frac{dy}{dt} = \frac{3}{2}(t+4)^{1/2}$
$L = \int_0^1 \sqrt{\frac{9}{4}t + \frac{9}{4}(t+4)}\,dt = \frac{3}{2}\int_0^1 (2t+4)^{1/2}\,dt = \frac{3}{2}\cdot\frac{1}{2}\cdot\frac{2}{3}(2t+4)^{3/2}\Big]_0^1 =$
$\frac{1}{2}\left(6^{3/2} - 4^{3/2}\right) = 3\sqrt{6} - 4$

D 26. $\int_0^2 (x^3 + 1)\,dx = \frac{1}{4}x^4 + x\Big]_0^2 = 4 + 2 - 0 = 6$

E 27. Minimize distance^2 from $(0,0)$ to $(x, \frac{8}{x})$.
$f(x) = x^2 + 64x^{-2}$ so $f'(x) = 2x - 128x^{-3}$ which equals zero when
$2x^{-3}(x^4 - 64) = 0 \Rightarrow x = 2\sqrt{2}$ (or $x = -2\sqrt{2}$)
Distance from $(0,0)$ to $(2\sqrt{2}, 2\sqrt{2})$ is 4.

A 28. $f'(x) = 1 - e^{-x}$ and $f''(x) = e^{-x}$
$f'(1) = 1 - 1/e$ $f''(1) = 1/e$ → both are positive so graph is increasing & concave up.

D 29. $\int_0^{\ln 2} \text{sech}\,x \tanh x\,dx = -\text{sech}\,x\Big]_0^{\ln 2} = -\frac{2}{e^x + e^{-x}}\Big]_0^{\ln 2} =$
$-\frac{2}{2 + 1/2} + \frac{1}{1+1} = -\frac{4}{5} + 1 = \frac{1}{5}$

C 30. $\begin{array}{r|rrr} 5 & 1 & -2 & 0 \\ & & 5 & 15 \\ \hline & 1 & 3 & 15 \end{array}$ so $y = x + 3 + \frac{15}{x-5}$. As $x \to \infty$, $y \to x + 3$
so the asymptote is $y = x + 3$.