Answers:
1. B
2. A
3. C
4. C
5. A
6. A
7. C
8. D
9. C
10. D
11. A
12. D
13. A
14. A
15. B
16. E
17. A
18. C
19. B
20. B
21. E
22. B
23. D
24. B
25. A
26. E
27. A
28. C
29. C
30. B

Solutions:

1. $\log\frac{x-2}{7-x}<1 \Rightarrow 0<\frac{x-2}{7-x}<10 \Rightarrow 2<x<\frac{72}{11}$

2. The expression is equivalent to $\sin(\frac{5\pi}{24}+\frac{\pi}{24})=\sin(\frac{\pi}{4})=\frac{\sqrt{2}}{2}$

3. $y=3\cos\left(2(x+\frac{\pi}{6})\right)+\frac{\pi}{2}$. Each change made to the function correctly corresponds to the changes desired in the graph.

4. $\mathbf{v}=40\cos(120^\circ)\mathbf{i}+40\sin(120^\circ)\mathbf{j}=-20\mathbf{i}+20\sqrt{3}\mathbf{j} \Rightarrow 20\sqrt{3}-20$

5. The projection of $\mathbf{u}$ onto $\mathbf{v}$ is $\left(\frac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v}$. Plugging in $\mathbf{u}=\langle 2,6\rangle$ and $\mathbf{v}=\langle 1,-2\rangle$, we find this is just $-2\mathbf{v}=\langle -2,4\rangle$

6. Note that $z_1=8\text{cis}\left(\frac{7\pi}{12}\right)$ and $z_2=4\text{cis}\left(\frac{3\pi}{12}\right)$. Using the properties of cis, it follows that

$$z_1z_2-\frac{z_1}{z_2}=(8)(4)\text{cis}\left(\frac{7\pi}{12}+\frac{3\pi}{12}\right)-\frac{8}{4}\text{cis}\left(\frac{7\pi}{12}-\frac{3\pi}{12}\right)=-16\sqrt{3}-1+(16-\sqrt{3})i$$

7

$$2\cos^3 x-7\cos^2 x+3\cos x=(\cos x)(2\cos x-1)(\cos x-3)=0\Rightarrow \cos x=\frac{1}{2},\cos x=0\Rightarrow 4 \text{ solutions}$$

8. Definition of a hyperbola

9. The monotonic quadratic with roots 1-2i and 1+2i is x^2-2x+5 . The cubic we seek is a constant multiple of $(x^2-2x+5)(x-3)=x^3-5x^2+11x-15$

10. Long division reveals that the slant asymptote is $y=x-1$. Therefore, the only asymptotes are $y=x-1$ and $x=3$.

11. This is a dimpled limaçon

12. This expression is the sum of the roots taken 4 at a time. From vieta's this is $\frac{e}{a}=\frac{-12}{4}=-3$

13. The third and last functions are even. Therefore the answer is -15.

14. $\cos(\frac{\pi}{3})=\frac{1}{2}$, and $\frac{\pi}{3}$ is in the traditional range for $\arccos(x)$.

15. From the law of cosines $14^2=15^2+13^2-2(13)(15)\cos\angle A\Rightarrow \cos\angle A=\frac{33}{65}$

16. Note that the first row is a constant multiple of the third, therefore the determinant is zero.

17. We add every point in A to every point in B as stated in the definition of the *Minkowski sum*.

18. Consider what happens to an arbitrary point (x,y) from set A when it undergoes the *Minkowski sum* function. This one specific point can be affected by any of the points on line segment B. In this sense, one can view set B as a vector, sliding every point of A along its line segment. In other words, *Minkowski sum* of two line segments is a parallelogram. The boundary points of this parallelogram can be intuitively found by adding the boundaries of line segments A and B. We find that the boundary points are (4,6), (10,6), (1,4), (7,4). The area of this parallelogram is (6)(2)=12.

19. The addition of a single point of A to every point on the circle acts as a translation of set B. Therefore, the *Minkowski sum* is two intersecting circles, both of radius 5, one centered at (2,4) and one centered at (5,8). The problem reduces to finding the area of two circles of radius 5 which intersect each other's center. This is 50π - the area of the intersection region. This intersection region can be viewed as two sectors of the circle minus two triangles. The intersection region has an area of $2\left(\frac{25}{3}\pi-\frac{25\sqrt{3}}{4}\right)$. It follows that the final answer is $\frac{100}{3}\pi+\frac{25\sqrt{3}}{2}$

20. Raising both sides of the equation as exponents of 8, we find that $(\log_4 x)^3=\log_{16}x$. Substituting $x=2^a$, we get $\left(\frac{1}{2}a\right)^3=\frac{1}{4}a\Rightarrow a=\sqrt{2}$ ($a=-\sqrt{2}$ is extraneous). This is closest to 1.5.

21. From Vieta's we have that the sum of the roots is zero $\Rightarrow a+b+c+d=0$. Therefore we can rewrite the expression as $-\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right)$. This is the negative sum of the reciprocal of the roots. It is known that reversing the coefficients of a polynomial results in a polynomial with reciprocal roots. Reversing the polynomial and using Vieta's, we find that the negative sum of the reciprocals is $-\frac{7}{5}$.

22. $r=\frac{8}{1+3\sin\theta}\Rightarrow r+3r\sin\theta=8\Rightarrow r=8-3y$. Squaring both sides and rearranging,

$$8y^2-48y+x^2+64=0\Rightarrow\frac{(y-3)^2}{1}-\frac{x^2}{8}=1\Rightarrow\frac{c}{a}=3$$

23. Dividing by 3, this expression becomes $\dfrac{\frac{\sqrt{3}}{3}+\tan 20^\circ}{1-\frac{\sqrt{3}}{3}\tan 20^\circ}=\tan(30+20)^\circ$.

It follows that the answer is 50°

24. For that specific configuration: $\frac{BD}{CD}=\frac{AB}{AC}\cdot\frac{\sin\angle BAD}{\sin\angle CAD}$.(this is a generalized version of the angle bisector theorem).This can be derived by applying law of sines carefully and equating angles with equal sines (IE: $\angle ADB$ and $\angle CDA$). Plugging in our numbers, we find that $\frac{\sin\angle BAD}{\sin\angle CAD}=\frac{21}{40}$

25. Consider the generalized relationship with x=$\frac{\pi}{17}$. Noting the symmetry, we try to apply sum to product to the numerator. It makes sense to combine the 5x and x, and the 4x and 2x:
$\sin(3x+2x)+\sin(3x-2x)=2\sin 3x\cos 2x$, $\sin(3x+x)+\sin(3x-x)=2\sin 3x\cos x$
Therefore the entire numerator is equal to $\sin 3x(1+2\cos x+2\cos 2x)$. Simplifying the denominator in the same fashion, we have that it is equal to $\cos 3x(1+2\cos x+2\cos 2x)$. Therefore, the entire fraction is $\tan(3x)$. Taking $x=\frac{\pi}{17}$, our answer is $\tan\frac{3\pi}{17}$.

26. $x^3+y^3+z^3-3xyz=(x+y+z)(x^2+y^2+z^2-yz-xz-xy)$. Since $x+y+z=0$.
$x^3+y^3+z^3=3xyz \Rightarrow \frac{x^3+y^3+z^3}{xyz}=3 \Rightarrow \frac{x^2}{yz}+\frac{y^2}{xz}+\frac{z^2}{xy}=3$ (this is always true, if $x+y+z$ =0)
Alternately, one can consider the expansion of $(x+y+z)^3$ and manipulate to find that
$x^3+y^3+z^3=3xyz$.

27. We find the distance from an arbitrary point (x,y) to the line and equate that to the distance between (x,y) and (2,3) :
$\frac{|4x+3y|}{\sqrt{4^2+3^2}}=\sqrt{(x-2)^2+(y-3)^2} \Rightarrow \frac{(4x+3y)^2}{25}=(x-2)^2+(y-3)^2$. Expanding and rearranging, we find that the locus is $9x^2-24xy+16y^2-100x-150y+325=0$

28. Let $z=r\text{cis}(\theta)=re^{i\theta}$, We will show that $\left|z+\frac{1}{z}\right|=\sqrt{r^2+\frac{1}{r^2}+2\cos 2\theta}$. We will prove this in two different ways.

Method 1: $\left|z+\frac{1}{z}\right|=\left|re^{i\theta}+\frac{1}{r}e^{-i\theta}\right|=\left|(r+\frac{1}{r})\cos\theta+(r-\frac{1}{r})i\sin\theta\right|=\sqrt{\left(r+\frac{1}{r}\right)^2\cos^2\theta+\left(r-\frac{1}{r}\right)^2\sin^2\theta}$
$=\sqrt{\left(r^2+\frac{1}{r^2}\right)(\cos^2\theta+\sin^2\theta)+2(\cos^2\theta-\sin^2\theta)}=\sqrt{r^2+\frac{1}{r^2}+2\cos 2\theta}$

Method 2: Consider the parallelogram formed in the complex plane by the origin, z , $\frac{1}{z}$, and $z+\frac{1}{z}$.
When one takes the reciprocal of a complex number, the angle associated with the complex number is

reflected about the x axis. Therefore, the parallelogram's angles are 2θ and $180-2\theta$. Now consider the triangle formed by the origin, z, and $z+\frac{1}{z}$. Applying law of cosines to this triangle, we find that

$$\left|z+\frac{1}{z}\right|=\sqrt{r^2+\frac{1}{r^2}+2\cos 2\theta}.$$

We now have that $1=r^2+\frac{1}{r^2}-2\cos 2\theta$. Applying trigonometric identities, we can rearrange to have:

$3r^2=r^4+1+4r^2\cos^2\theta \Rightarrow (r\cos\theta)^2=(\Re(z))^2=\frac{-r^4+3r^2-1}{4}$. Since this is a quadratic in r^2, we can maximize the real part of z. We find that $r^2=\frac{3}{2}\Rightarrow \Re(z)=\frac{\sqrt{5}}{4}$.

29. We will generalize the result. Let $F_1P=x$ and $F_2P=y$. Note that $x+y=2a$ by the definition of an ellipse. Let $\angle F_1PF_2=\theta$. From the Law of Cosines: $4c^2=x^2+y^2-2xy\cos\theta$. From our note earlier, we have that $x^2+2xy+y^2=4a^2$. Also we know that $A=\frac{1}{2}xy\sin\theta=144$. Plugging in both of these to our expression, we have that

$$4c^2=(4a^2-2xy)-2xy\cos\theta=4a^2-2xy(\cos\theta+1)=4a^2-\frac{576}{\sin\theta}(\cos\theta+1)$$

Rearranging, we have $4a^2-4c^2=576\frac{\cos\theta+1}{\sin\theta}\Rightarrow\frac{\sin\theta}{\cos\theta+1}=\frac{144}{b^2}\Rightarrow\tan\frac{\theta}{2}=\frac{1}{4}$. Applying the double angle formula, we have that $\tan\theta=\frac{8}{15}$. (Note: Alternate solution exists with Heron's instead of Law of Cosines).

30. We note that $\sin^3 B+\cos^3 C=(\sin B+\sin C)(\sin^2 B-\sin B\sin C+\sin^2 C)=1$. The second expression reminds us of law of cosines, so we have that $a^2=b^2+c^2-2bc\cos A=b^2+c^2-bc$. Note that this is very similar to our expression, but with side lengths instead of sines. We look to the law of sines, and realize that $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}=k$. Therefore, if we multiply the equation through by $\frac{1}{k^2}$, we can substitute this for sines: $\sin^2 A=\sin^2 B-\sin B\sin C+\sin^2 C=\frac{3}{4}$. We now have that

$\sin B+\sin C=\frac{4}{3}$. Applying sum to product, we have $\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)=\frac{2}{3}$

Therefore, $\cos\frac{B-C}{2}=\frac{4\sqrt{3}}{9}$. Applying the double angle formula, we find that $\cos(B-C)=\frac{5}{27}$