January 2005 *Invitational Algebra 1 Team solutions*

1. $A = 35x^2 - 3x - 3 \rightarrow B = -15x^2 - x + 2 \rightarrow C = 63x^2 + 17x - 10 \rightarrow D = 15x^2 + 17x - 4$

$= 98x^2 + 30x - 14 - 48x^2 = 50x^2 + 30x - 14$

2. $7 + 11 + (-1) + (-3) + 2 = 16$

3. $4m^2 + 4m - 15 = 0 \rightarrow (2m + 5)(2m - 3) = 0 \rightarrow \frac{-5}{2} + \frac{3}{2} = -1$

$8d^2 - 6d - 5 = 0 \rightarrow (4d - 5)(2d + 1) = 0 \rightarrow \frac{5}{4} - \frac{1}{2} = \frac{3}{4}$

$4w^2 - 11w + 6 = 0 \rightarrow (4w - 3)(w - 2) = 0 \rightarrow \frac{3}{4} \bullet 2 = \frac{3}{2}$

$3n^2 - 10n + 8 = 0 \rightarrow (3n - 4)(n - 2) = 0 \rightarrow \frac{4}{3} \bullet 2 = \frac{8}{3}$

$(-1)\left(\frac{3}{4}\right) + \frac{3}{2} \div \frac{8}{3} = \frac{-3}{4} + \frac{3}{2} \bullet \frac{3}{8} = \frac{-3}{4} + \frac{9}{16} = \frac{-3}{16}$

4. $Q = 2, G = 13, P = 156, and\ H = 97 \rightarrow \frac{156 - 13}{97} \bullet \frac{-156}{3} \bullet \frac{2}{143} = \frac{143}{97} \bullet -52 \bullet \frac{2}{143} = \frac{-104}{97}$

5. $A = -7 + 8 = 1, B = 45 - 49 = -4, C = 8 - 3 - 7 = -2 \rightarrow \frac{4 + \sqrt{16 - 4 \bullet 1 \bullet -2}}{2}$

$= \frac{4 + \sqrt{24}}{2} = \frac{4 + 2\sqrt{6}}{2} = 2 + \sqrt{6}$

6. $r\left(\frac{d}{3} + \frac{d}{5}\right) = 2d \rightarrow r\left(\frac{8d}{15}\right) = 2d \rightarrow \frac{8r}{15} = 2 \rightarrow r = \frac{15}{4}$

7. $3 + 2 - 8 + 5 - 9 = -7 \rightarrow 8(-2)^5 + 4(-2)^3 - 2(-2) + 3 = -281 \rightarrow (-7)(-281) = 1967$

8. $200-20-(62-20)-(58-20)=100 \rightarrow \frac{100}{200}=\frac{1}{2} \rightarrow 2 \bullet \frac{1}{2}=1$

9. $A=\frac{7! \bullet 8!}{9 \bullet 8!} \bullet \frac{1}{56}=\frac{7 \bullet 6 \bullet 5 \bullet 4 \bullet 3 \bullet 2}{9 \bullet 56}=\frac{30}{3}=10, B=\frac{9 \bullet 8 \bullet 7 \bullet 6! \bullet 3!}{6!} \bullet \frac{1}{336}=\frac{9 \bullet 8 \bullet 7 \bullet 3 \bullet 2}{336}=9,$

$\frac{10}{9}=\frac{C}{18} \rightarrow 9C=10(18) \rightarrow C=20,(10,20)(0,9) \rightarrow D=\sqrt{10^2+11^2}=\sqrt{100+121}=\sqrt{221}$

10. $\frac{10}{7}=.142857 \textit{ repeating} \rightarrow \frac{49}{6}=8 \textit{ with remainder of } 1 \rightarrow 1$

11. $X=\left(\frac{26+32}{2} \bullet \frac{14+6}{2}\right)=(29,10) \rightarrow Y=5(3y+9)+20=2y \rightarrow 15y+45y+20=2y$

$13y=-65 \rightarrow y=-5 \rightarrow -6=3(-5)+9=x \rightarrow (-6,-5) \rightarrow Z \rightarrow \frac{-5-10}{-6-29}=\frac{3}{7}$

$3x-7y=c \textit{ if perpendicular } 7x+3y=c \rightarrow -42-15=-57=c \rightarrow 7x+3y=-57$

12. $3x^2-3x+1=4x^2-12x+9 \rightarrow x^2-9x+8=0 \rightarrow (x-8)(x-1)=0 \rightarrow x=8 \textit{ or } 1$

$\textit{One is extraneous} \rightarrow x=8$

13. $\textit{I – Sum of cubes} \rightarrow \textit{II – Factor out } x \rightarrow \textit{III – Factor out } 3 \rightarrow \textit{V – Factor out } 3$

$\textit{IV – Is only prime}$

14. *II and IV are true*

15. $\sqrt{256+\frac{1}{256}}=\sqrt{256+2+\frac{1}{256}}=\sqrt{16^2+2+\left(\frac{1}{16}\right)^2}=\sqrt{16^2+2(16)\left(\frac{1}{16}\right)+\left(\frac{1}{16}\right)^2}$

$\sqrt{\left(16+\frac{1}{16}\right)^2}=16+\frac{1}{16}=16\frac{1}{16}$