Algebra 2 Team Solutions Berkeley Invitational 1999

$$8^{1/6}+x^{1/3}=\frac{7}{3-\sqrt{2}}\bullet\frac{3+\sqrt{2}}{3+\sqrt{2}}$$

1. A = 4; B = 8; $\sqrt{2}+x^{1/3}=3+\sqrt{2}$ D: slope of the ⊥ line is –5.

$$x^{1/3}=3\Rightarrow x=27=C$$

A + B – C + D = 4 + 8 –27 + -5 = -20

2. $x^2-6x+y^2+4y=12\Rightarrow(x-3)^2+(y+2)^2=25$
The distance from center of the circle to (7, y) = 5, which yields y = -5 or 1. We use y = -5. The slope of the radius is - ¾ , so the slope of the tangent line is 4/3. Using the point (7, -5) and the slope, we get $y=\frac{4}{3}x-\frac{43}{3}$

3. $\log_8 3=m\Rightarrow 8^m=3$. Also, $\log_3 5=n\Rightarrow 3^n=5\Rightarrow 3=5^{1/n}$.
Combining the 2 equations gives $2^{3m}=5^{1/n}\Rightarrow 2=5^{1/3mn}$ and $10=5^{1+(1/3mn)}$. Taking the log of both sides and solving for log5 gives $\log 5=\frac{1}{1+(1/3mn)}=\frac{3mn}{3mn+1}$

4. $\frac{7.2}{x}+\frac{7.2}{y}=1$ and $\frac{2}{x}+\frac{6}{x}+\frac{6}{y}=1$

Solving, yields x = 12 and y = 18. Using 5 hrs. with x and 9 hrs. with y, gives $\frac{5}{12}+\frac{9}{18}=\frac{11}{12}$.

So, $\left(1-\frac{11}{12}\right)$(# of gallons) = 20 and gallons = 240.

5. $\frac{416-41}{900}=\frac{5}{12}\Rightarrow 5+12=17$

6. $f(1)=1$ and $f(2)=1$. Substituting in the formula gives $f(3)=2,\ f(4)=3,\ f(5)=5,\ldots$with each new function equaling the sum of the 2 preceding ones (or one can use the formula for each term). Hence, $f(12)=144$.

7. $\frac{(x+h)^2+3(x+h)+3-x^2-3x-3}{h}=2x+h+3.$

8. $\left(\frac{x^{a+4}}{x^{-4a-1}}\right)\left(x^{2a}\right)=x^{3587}\Rightarrow\left(x^{5a+5}\right)\left(x^{2a}\right)=x^{3587}$

$7a+5=3587\Rightarrow a=\frac{3582}{7}$

9. $9^x - 9^{x-1} = 24 \Rightarrow 9^x(1-9^{-1}) = 24$
$9^x = 27 \Rightarrow x = \underline{\underline{1.5}}$

10. 0.80(15) = 12 and 0.40(12) = 4.8
12 – 4.8 = 7.2 and 15 – 7.2 = 7.8
12 + 4.8 + 7.8 = <u>24.6 miles</u>.

11. A: $f^{-1}(x) = \frac{4}{9}f(x) \Rightarrow x = \frac{4}{9}f(x)$

$f(x) = \frac{9}{4}x \Rightarrow f\left(\frac{9}{4}\right) = \left(\frac{9}{4}\right)^2 = \frac{81}{16}$

B: ${}_5C_3\left(a^{-1/2}\right)^2\left(\frac{-2}{5}a^2\right)^3 = 10a^{-1}\left(\frac{-8}{125}a^6\right) = \frac{-80}{125}a^5 \Rightarrow$ which gives a coefficient of $\frac{-16}{25}$

So, $A \bullet B = \frac{81}{16} \bullet \frac{-16}{25} = \underline{\underline{\frac{-81}{25}}}$

12. Using $x^2 + 7x + 1 = 0$ gives2 negative roots: $x = \frac{-7 \pm \sqrt{45}}{2}$.

Using $x^2 + 5x + 2 = 0$ gives2 negative roots: $x = \frac{-5 \pm \sqrt{17}}{2}$.

Using $x^2 + 5x + 4.5 = 0$ gives2 negative roots: $x = \frac{-5 \pm \sqrt{7}}{2}$.

Adding: $\frac{-7+\sqrt{45}-7-\sqrt{45}-5+\sqrt{17}-5-\sqrt{17}-5+\sqrt{7}-5-\sqrt{7}}{2} = \underline{\underline{-17}}$

13. log 158766125 - log 805 = log 197225 (197225 = $23 \bullet 7^3 \bullet 5^2$)
This yields log 7^3 + log 5^2 + log 23 = <u>3a +2c +b.</u>

14. $(10-x)12 = (10+x)9 \Rightarrow x = \frac{30}{21} = \underline{\underline{\frac{10}{7}}}$.

15. Let x be the sides of the rain gutter. A = (512 – x)(x). The maximum occurs at $\frac{-b}{2a}$ = <u>256</u>.