

February Regional Calculus Team Solutions

1) $57 + \frac{\sqrt{3}}{6}$

A = $\frac{\sqrt{3}}{6}$, factor out $\sqrt{x} - \sqrt{3}$ from top and bottom and plug in $x=3$

B = 56, dividing by $x-1$ leaves $(x-8)(x-9)$ on top and plug in $x=1$

C = 2, just plug in x value

D = 1, multiply top and bottom by $\sqrt{x^2 + 2x} + x$ and take the limit as x goes to infinity

2) **7419**

A = 11, $f'(x) = 2x+7$, then plug in $x=2$

B = 36, $g''(x) = 6x$, then plug in $x=6$

C = 5324, derivative of $f(g(x)) = f'(g(x))(g'(x))$ which is $(2(x^3 - 4x + 9) + 7)(3x^2 - 4)$, then plug in $x=4$

D = 2048, the second derivative is $30x^6 - 96x^2 + 150x + 32$, then plug in $x=3$

3) **$9\pi - 1$**

$f'(x) = \frac{x(3^x \pi \sec^2(\pi x) + 3^x \ln(3) \tan(\pi x)) - 3^x \tan(\pi x)}{x^2}$ and $f'(3) = \frac{3(27\pi)}{9} = 9\pi$

$g'(x) = \sec^2(x)$ because $g(x)$ is just $\tan(x)$, so $g'(0) = 1$

4) $62 + \frac{2\sqrt{33}}{3}$

A = **6**, acceleration is $6t-24$, then plug in $t=5$

B = **-32**, at $t=0$, $s(0) = -3$ and at $t=2$, $s(2) = 8-48+8-3 = -35$. So displacement is $-35 - (-3) = -32$

C = **-28**, average rate of change is $\frac{s(4) - s(0)}{4 - 0}$ which is $\frac{-115 - (-3)}{4}$

D = **$4 - \frac{2\sqrt{33}}{3}$** velocity equals $3t^2 - 24t + 4$, then obtain the smaller root via quadratic formula

5) **220**

A = 286, right hand Riemann sum is $2(23+45+75)$

B = 154, left hand Riemann sum is $2(9+23+45)$

C = 220, trapezoidal rule is $1(9+2(23)+2(45)+75)$

6) **(1,3)**

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Every point on the graph of the given function can be expressed as (x, x^2+2) . So, the distance to the point $(5,1)$ can be expressed as $\sqrt{(x-5)^2 + (x^2+1)^2} = \sqrt{x^4 + 3x^2 - 10x + 26}$. To minimize the distance, we must set its derivative equal to 0. The derivative of the distance formula is $\frac{4x^3+6x-10}{2\sqrt{x^4+3x^2-10x+26}}$. So, setting the numerator equal to 0, the only real root we get is $x=1$, so the point on the graph closest to $(5,1)$ is $(1, 1^2 + 2) = (1,3)$

7) 3456

B= 2, set $\frac{2x-(-3)}{4}$ equal to $2x+4$, solve for x

A= 36, this is simply $11+9+7+5+3+1$

R= $\frac{1}{2}$, take the derivative to get $3x^2 - \frac{3x}{2}$ and solve for x to get $\frac{1}{2}$ and 0, plug in these two and other endpoint to see which is the minimum

Y= 192, 8 minutes after flying 240 mph, the plane has traveled 32 miles, so the distance between the person and the plane is $\sqrt{24^2 + 32^2}=40$ so $x^2 + y^2 = z^2$ and take the derivative to get $x dx + y dy = z dz$ and dx equal 0 so $40 dz = (32)(240)$

8) $\frac{3\pi}{2} + \frac{7}{4}$

A= 1, derivative is just $\tan(x)$ and at $x = \frac{\pi}{4}$, the slope is just 1

B= $\frac{3\pi}{2}$, this is just one quarter of the area of the ellipse $4y^2 + 9x^2 = 36$

C= 1, take the derivative, giving $2x-2$, and set equal to zero

D= $-\frac{1}{4}$, taking the derivative twice and solving for solutions yields no solutions between 4 and 6, so just plug in endpoints to slope

9) 217

A= 10, the answer is infinite

B= 10, the answer is infinite

C= 169, derivative is $18x^2 + 7$, then plug in $x=3$

D= 28, integrate from 0 to 3 and divide by three

10) 35

A is FALSE because the limit taken from the right side is 2, not 4

B is FALSE because the limit taken from the left side is 4, not 2

C is FALSE because at $x=3$, $f'(x)$ is defined and is -1

D is TRUE because at $x=1$ the limit from the left does not match that of the limit from the right, so the function is not continuous at $x=1$

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11) -12.5

A = $\frac{3}{4}$, the derivative is $\frac{-g'(x)}{(g(x))^2}$, then plug in the values of $g(1)$ and $g'(1)$

B = 0, the derivative is $f'(g(x))g'(x)$, then plug in the values for $x=2$ for $g(x)$, which is 3, and $f'(3)$ is 0

C = -10, the derivative is $f'(x)g(x) + g'(x)f(x)$, then plug in the values at $x=3$

D = $\frac{-13}{4}$, the derivative is $\frac{g'(x)f'(x) - f'(x)g'(x)}{(f(x))^2}$, then plug in the values at $x=0$

12) $1 + \frac{\pi}{4}$

The function inside the integral is the derivative of $\tan^{-1} t$. So, integrating this derivative from 0 to $\tan(x)$ gives us $\tan^{-1} \tan(x)$, which, from

$-\frac{\pi}{2} < x < \frac{\pi}{2}$, is x , $F(\frac{\pi}{4})$ is just $\frac{\pi}{4}$, and $F'(\frac{\pi}{4})$ according to the second

fundamental theorem of calculus is $\frac{\sec^2(\frac{\pi}{4})}{1 + \tan^2(\frac{\pi}{4})} = 1$

13) $2^{2012} \sin(2x)$

the n^{th} derivative of $\sin(2x)$ will have a leading coefficient of 2^n and since n is divisible by 4, then the function following the coefficient will be $\sin(2x)$

14) $\frac{8\sqrt{3}}{9}$

the diagonal of the cube is $2r$, which is 2, and the diagonal is $s\sqrt{3}$ so $s = \frac{2}{\sqrt{3}}$

and $V = s^3$ which is $\frac{8\sqrt{3}}{9}$

15) 2

the derivative of an odd function is an even function, so $f'(1) = f'(-1)$