ANSWERS AND SOLUTIONS

1. **(C)** 5

   We have Chris, Kevin, and 4 mentioned siblings. However, one of the brothers that Kevin has is Chris. Thus, there are a total of 5 siblings.

2. **(D)** 50

   There are $\frac{1200 \cdot 5}{30} = 200$ classes a day. $\frac{200 \text{ classes}}{4 \frac{\text{classes}}{\text{teacher}}} = 50$ teachers.

3. **(B)** 4

   We need all integers such that $7x + 2$ is divisible by 3. Note that $7x + 2 = 6x + (x + 2)$, and $6x$ is divisible by 3. Therefore, $x + 2$ must be divisible by 3. Thus, $x = ... - 5, -2, 1, 4, 7...$ and can be expressed in the form $3n + 1$. $3 + 1 = 4$.

4. **(D)** $5 + \sqrt{24}$

   Using FOIL, we obtain $2 + \sqrt{6} + \sqrt{6} + 3 = 5 + 2\sqrt{6} = 5 + \sqrt{4} \cdot \sqrt{6} = 5 + \sqrt{24}$.

5. **(D)** 36

   By the Power of a Point theorem, $AE \cdot CE = BE \cdot DE$. Thus, $xy = 6 \cdot 6 = 36$.

6. **(E)** NOTA

   From the previous problem, we know that $xy = 36$. If AC passes through the center of the circle, it is a diameter (the longest possible chord). Thus, $x + y \leq 12$, and the only solution is $x = y = 6$, giving $x + y = 12$. There is only 1 possible value of $x + y$.

7. **(C)** Mr. Snow

   One approach is to consider when each sponsor is guilty, and evaluate the 4 statements:
   Mrs. Funk → F, T, T, T → 3 true statements
   Dr. Morris → F, F, F, T → 1 true statement
   Mr. Snow → T, T, F, F → 2 true statements
   Mr. Wiggins → T, T, F, T → 3 true statements
   There are exactly two true statements only when Mr. Snow is guilty.

8. **(A)** Dhyan, 3

   Note that a must be rounded to the nearest integer. Dhyan's tennis function value is $89 + 82 + 80 = 251$, while Patrick's is $63 + 95 + 90 = 248$. Dhyan is better by $251 - 248 = 3$.

9. **(D)** 97

Note that $\sum = 173 + 9s - a$. Clearly, we wish to minimize s and maximize a. Since $\sum = a + p + 10s$, it is most important to minimize s, so let $s = 1$. Then, $\sum = 182 - a$ and $2a + p = 172$. Since a and p must be positive integers, let $p = 2$ to maximize a at $\frac{172-2}{2} = 85$. Thus, the minimum $\sum$ is $182 - 85 = 97$.

10. **(E)** NOTA

The radical sign denotes the principal square root of the radicand. When x is negative, $\sqrt{x^2} = -x$ and when x is positive, $\sqrt{x^2} = x$. More concisely, $\sqrt{x^2} = |x|$. No choice given is equivalent to this.

11. **(B)** 8%

Let the original length and width be l and w, respectively. Therefore, the new length and width are $.9l$ and $1.2w$, and the new area is $1.08lw$. The percent change is $1.08 - 1 = .08 = 8\%$.

12. **(C)** 3

As the number of sides of a regular polygon increases, the shape becomes nearly a circle. The area is approximately $\pi \cdot 1^2 = \pi$, which is closest to 3.

13. **(A)** Daniel, 1

One approach is to consider all four cases:
If Daniel takes 1 stone, Pratik will be forced to leave either 1 or 2 stones, and Daniel will win.
If Daniel takes 2 stones, Pratik will take 4 and win.
If Pratik takes 3 stones, Daniel will take 1 or 2; Pratik cannot move, and Daniel will take the remaining stone(s) and win.
If Pratik takes 4 stones, Daniel will take 2 and win.
Only when Daniel goes first and takes 1 stone will the first player to move win.

14. **(D)** 24

Since $\sqrt{\sqrt{xyz}} = \sqrt{\sqrt{x}} \cdot \sqrt{\sqrt{y}} \cdot \sqrt{\sqrt{z}}$, we have $2^2 \cdot 3 \cdot \sqrt{4} = 4 \cdot 3 \cdot 2 = 24$.

15. **(B)** 9

Since $AC < AB$, C must be on the same side as A from B. To minimize BC, C should be between A and B, so $BC = 12 - 3 = 9$.

16. **(C)** $2b - a + 1$

Note that $245 = 7^2 \cdot 5$, so $\log 245 = 2\log 7 + \log 5$. Also, $\log 5 = \log(\frac{10}{2}) = \log 10 - \log 2 = 1 - a$. Thus, $\log 245 = 2b + (1 - a) = 2b - a + 1$.

17. **(B)** $2\sqrt{3}$

The area of a hexagon is the composition of 6 equilateral triangles with area $\frac{s^2\sqrt{3}}{4}$, or $\frac{3s^2\sqrt{3}}{2}$. So, $\frac{3s^2\sqrt{3}}{2} = 24\sqrt{3}$, and $s = 4$. The radius is the height of one of these triangles, or $\frac{4\sqrt{3}}{2} = 2\sqrt{3}$.

18. **(B)** 180

This is a right triangle, so the area is $\frac{1}{2} \cdot 9 \cdot 40 = 180$. If this was not recognized, Heron's Formula gives $\sqrt{45 \cdot 4 \cdot 5 \cdot 36} = 3 \cdot 5 \cdot 2 \cdot 6 = 180$.

19. **(B)** 25

If they sell x frisbees, they will have a revenue of $x(10 - .02x) = 10x - .02x^2$. However, it will cost them $100 + 9x$, so the profit is $(10x - .02x^2) - (9x - 500) = -.02x^2 + x + 500$. This is a downward pointing parabola that is maximized at its vertex which occurs at $\frac{-1}{-.04} = 25$.

20. **(A)** 12π

The volume of a cone is $\frac{1}{3}r^2h$. The radius is $\frac{6}{2} = 3$, giving a volume of $\frac{1}{3} \cdot 9 \cdot 4 = 12\pi$.

21. **(B)** 61.2%

The Juicy-Juice contributes $30 \cdot 1 = 30$ ounces of juice, while the Kool-Aid contributes $20 \cdot .03 = .6$ ounces of juice. The fraction is $\frac{30+.6}{50} = \frac{60+1.2}{100} = 61.2\%$.

22. **(C)** 3

By the definition of an inverse, to evaluate $f^{-1}(a)$, we are looking for the value of b such that $f(b) = a$. Since $f(3) = 2$, $f^{-1}(2) = 3$. $h(3) = 4$, and since $g(2) = 4$, $g^{-1}(4) = 2$. Finally, $f(2) = 3$.

23. **(B)** 22

Use that $A = \frac{h(b_1+b_2)}{2}$ and note that the heights are 1 to obtain $\frac{0+1}{2} + \frac{1+4}{2} + \frac{4+9}{2} + \frac{9+16}{2} = 22$.

24. **(A)** $M > N$

A rough drawing of this graph shows that the parabola "curves below" the straight edges of the trapezoids. Thus, the estimation overestimates the area, and $M > N$. In fact, $N = \frac{64}{3} \approx 21.3$.

25. **(D)** 51

The number of digits will be one more than the highest power of 100 that is less than or equal to 10^{100}. Note that $10^{100} = (10^2)^{50} = 100^{50}$, so the number of digits is $50 + 1 = 51$.

26. **(C)** 1189

To create symmetry, let $x = 34.5$. Then, $\sqrt{(x-1.5)(x-.5)(x+.5)(x+1.5)+1} = \sqrt{x^4 - \frac{5}{2}x^2 + \frac{25}{16}}$ $= \sqrt{(x^2 - \frac{5}{4})^2} = x^2 - \frac{5}{4} = 1190.25 - 1.25 = 1189$. There are several other creative approaches.

27. **(A)** -1

Since 1 is a root, $f(1) = 0 = 1 + a + b + c + d + e$. Thus, $a + b + c + d + e = 0 - 1 = -1$.

28. **(B)** 8

We have that side length $= \frac{k}{n}$. A square with area 4 has side length 2, so $2 = \frac{k}{4}$ and $k = 8$. The hexagon has side length $\frac{8}{6}$ and perimeter $6 \cdot \frac{8}{6} = 8$.

29. **(D)** 116

Rewrite this equation as $ER - 29E - 29R = -800$. So, $(E-29)(R-29) = -800 + 29^2 = 41$. Since 41 is prime, the two factors are either 1 and 41 or -1 and -41. Because we desire the value of $E + R$, we do not care which is which. For the first case, $E = 1 + 29 = 30$ and $R = 29 + 41 = 70$, so $E + R = 100$. For the second case, $E = 29 - 1 = 28$ and $R = 29 - 41 = -12$, so $E + R = 16$. The sum of the two possible sums is $100 + 16 = 116$.

30. **(A)** Euclid

Elements was written by Euclid; if you lived a few centuries ago, it would have been your textbook.