1. The sum of the angles of a triangle is $180°$. $180° - 40° - 80° = 60°$. $\boxed{\text{B}}$

2. The obtuse angle is supplementary to $70°$. $180° - 70° = 110°$. $\boxed{\text{A}}$

3. This is a theorem. $\boxed{\text{A}}$

4. Let $AB = 3x$, $BC = 5x$. Then $AC = 3x + 5x = 8x$, so $AB : AC = 3x : 8x = 3 : 8$. $\boxed{\text{A}}$

5. The contrapositive of the given statement is "If it's not here, then it's not hot." This is equivalent to $\boxed{\text{B}}$.

6. All are congruency theorems. $\boxed{\text{E}}$

7. The other angle is $180° - 35° - 110° = 35°$. Two angles are equal, so the triangle is isosceles. $110°$ is greater than $90°$, so the triangle is obtuse. $\boxed{\text{B}}$

8. $\angle B + \angle C = 180° - 120° = 60°$. $\angle B$ and $\angle C$ are both positive, so $\angle B > 0°$. Also, $\angle B < 60°$, or else $\angle C$ would be $0°$ or negative. $\boxed{\text{A}}$

9. Since $\angle DBA + \angle DBC = 180°$, choice (A) would tell you $2\angle DBA = 180° \Rightarrow \angle DBA = 90°$. Choice (B) implies (A), and choice (C) implies (B) by SSS congruency. Choice (D) does not give you enough information to conclude $\angle DBA = 90°$ (the two facts are redundant and both tell you that $\triangle DAC$ is isosceles, but B could be any point on AC). $\boxed{\text{D}}$

10. Scale down the triangle by a factor of 2009 so the legs have lengths 3 and 4. By the Pythagorean Theorem, the hypotenuse is $\sqrt{3^2 + 4^2} = 5$. Scaling back up, $AC = 5 \cdot 2009$. $\boxed{\text{B}}$

11. The triangle formed by two adjacent sides of the square and a diagonal is a $45°$-$45°$-$90°$ triangle with hypotenuse d. The side of the square is $\frac{d}{\sqrt{2}}$, so the area is $(\frac{d}{\sqrt{2}})^2 = \frac{d^2}{2}$. $\boxed{\text{B}}$

12. Let the two smaller side lengths be a and b. By the triangle inequality, $a + b > 50$. But a and b are even integers, so $a + b$ must also be an even integer. So the smallest possible value of $a + b$ is 52, and the smallest possible perimeter is $52 + 50 = 102$. This can be achieved by a triangle with side lengths 26, 26, 50. $\boxed{\text{D}}$

13. Draw an altitude to form a $30°$-$60°$-$90°$ triangle with hypotenuse 6, and the altitude opposite the $60°$ angle. The altitude has length $3\sqrt{3}$. $\boxed{\text{C}}$

14. All are sufficient except choice (C). Any isosceles trapezoid would satisfy (C). $\boxed{\text{C}}$

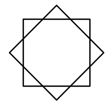
15. By AA similarity, $\triangle ABC \sim \triangle ACD$. So $\frac{AB}{AC} = \frac{AC}{AD} \Rightarrow \frac{4}{6} = \frac{6}{AD} \Rightarrow AD = 9$. $\boxed{\text{D}}$

16. Each of the sides of the original 20-gon is counted once, and the drawn diagonal is counted twice (it's a side of both the m- and n-gons). $20 + 2 = 22$. $\boxed{\text{C}}$

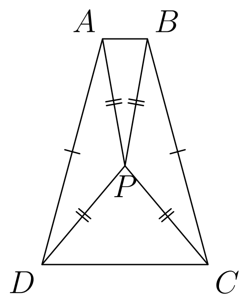
17. $\angle ABE = 180° - 110° - 30° = 40°$, so $\angle ABC = \angle ABE + \angle DBC = 40° + 60° = 100°$. $AB \,||\, CD$, so $\angle ABC$ and $\angle BCD$ are supplementary, so $\angle BCD = 180° - \angle ABC = 180° - 100° = 80°$. $\boxed{\text{B}}$

18. The second picture is a pentagon, whose sum of interior angles is $(5 - 2)180° = 540°$. The three unmarked angles are right and have sum $3 \cdot 90° = 270°$, so the sum of the marked angles is $540° - 270° = 270°$. $\boxed{\text{D}}$

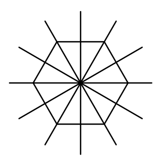
19. One side of a square can intersect another square in at most 2 places, so the four sides of a square can intersect another square in at most 8 places. An example is shown below. $\boxed{\text{D}}$



20. By SSS congruency, $\triangle APD \cong \triangle BPC$. Thus, $\angle APD = \angle BPC$. $\angle APD + \angle BPC = 360° - \angle APB - \angle CPD = 360° - 20° - 80° = 260°$, and since $\angle APD = \angle BPC$, $\angle APD = \frac{1}{2} \cdot 260° = 130°$. $\triangle APD$ is isosceles, so $\angle PAD = \frac{1}{2}(180° - 130°) = 25°$. $\triangle APB$ is isosceles, so $\angle PAB = \frac{1}{2}(180° - 20°) = 80°$. So $\angle BAD = \angle PAB + \angle PAD = 80° + 25° = 105°$. $\boxed{\text{C}}$



21. There are six, as shown below. $\boxed{\text{B}}$



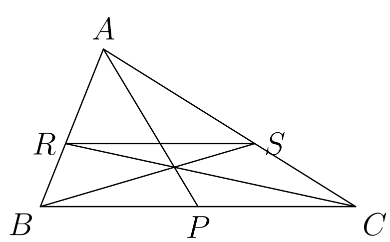
22. The quadrilaterals cover the area of triangle ABC twice. $2 \cdot 12 = 24$. $\boxed{\text{C}}$

23. The exterior angle is $\frac{360°}{n}$, so $\frac{360°}{n} > 10° \Rightarrow n < 36$. There are 35 positive integers below 36, but we have to exclude 1 and 2 because $n \geq 3$ and a polygon can't have 1 or 2 sides. $35 - 2 = 33$. $\boxed{\text{C}}$

24. Applying Ceva's Theorem, $\frac{1}{ZB} \cdot \frac{3}{2} \cdot \frac{4}{3} \Rightarrow ZB = 2$. $\boxed{\text{D}}$

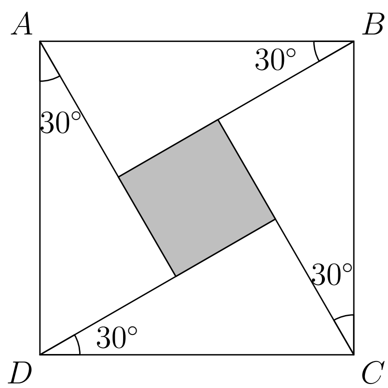
25. X, Y, and Z are the midpoints of the sides, so AX, BY, and CZ are medians. $\boxed{\text{A}}$

26. By Ceva's Theorem, $\frac{AR}{RB} \cdot \frac{BP}{PC} \cdot \frac{CS}{SA} = 1$. $\frac{AR}{RB} = \frac{3}{2}$, and $\frac{CS}{SA} = \frac{BR}{RA} = \frac{2}{3}$, so this becomes $\frac{3}{2} \cdot \frac{BP}{PC} \cdot \frac{2}{3} = 1 \Rightarrow \frac{BP}{PC} = 1 \Rightarrow BP = PC$. So P is the midpoint of BC. By similar triangles, $\frac{AR}{RS} = \frac{AB}{BC} \Rightarrow \frac{3}{6} = \frac{5}{BC} \Rightarrow BC = 10$. Since P is the midpoint of BC, $BP = \frac{1}{2} \cdot 10 = 5$. $\boxed{\text{D}}$



27. By the Pythagorean Theorem, $BC = \sqrt{10^2 - 7^2} = \sqrt{51}$. Since $7 = \sqrt{49} < \sqrt{51}$, and the hypotenuse is larger than either leg, AB is the smallest side of the triangle, so the angle opposite it, $\angle C$, is the smallest angle. $\boxed{\text{C}}$

28. The side length of the square is $\sqrt{16} = 4$. Divide the unshaded region into the 4 triangles shown below. Each triangle is a 30°-60°-90° triangle with hypotenuse 4, so the legs are 2 and $2\sqrt{3}$. The area of each traingle is $\frac{1}{2} \cdot 2 \cdot 2\sqrt{3} = 2\sqrt{3}$, so the area of the unshaded region is $4 \cdot 2\sqrt{3} = 8\sqrt{3}$. The area of the shaded region is the area of the square minus the unshaded region, or $16 - 8\sqrt{3}$. $\boxed{\text{B}}$



29. Let the interior angle measures of a regular m-gon and n-gon be x and y, respectively, so $x = \frac{k}{2} \cdot y$. Since $x > y$ (because $m > n$), k must be at least 3. Therefore, x is at least 3/2 times y.

    If $y \geq 120°$, then $x \geq \frac{3}{2} \cdot 120° = 180°$. However, x must be less than 180° because it is the interior angle measure of a regular polygon. Therefore, we can't have $y \geq 120°$, so $y < 120°$. This restricts y to 60°, 90°, and 108°, corresponding to $n = 3$, 4, and 5, respectively.

    Suppose $y = 60°$. If $k = 3$, then $x = 90°$, so $m = 4$. If $k = 4$, then $x = 120°$, so $m = 6$. If $k = 5$, then $x = 150°$, so $m = 12$ (the exterior angle is 30°, so $m = \frac{360°}{30°} = 12$). k can't be any larger or else $x \geq 180°$, so there are 3 solutions for this case.

    Suppose $y = 90°$. If $k = 3$, then $x = 135°$, so $m = 8$. k can't be any larger or else $x \geq 180°$, so there is 1 solution for this case.

    Suppose $y = 108°$. If $k = 3$, then $x = 162°$, so $m = 20$ (the exterior angle is 18°, so $m = \frac{360°}{18°} = 20$). k can't be any larger or else $x \geq 180°$, so there is 1 solution for this case.

    This gives us a total of $3 + 1 + 1 = 5$ solutions. $\boxed{\text{B}}$

30. Triangles ABC, CDE, and EFA are each isosceles triangles with angles $100°$, $40°$, and $40°$, so they are all similar to each other. So, $AC : CE : EA = AB : CD : EF = 6 : 3 : 3\sqrt{3} = 2 : 1 : \sqrt{3}$. Thus, triangle ACE is a $30°$-$60°$-$90°$ triangle, with $\angle EAC = 30°$. So, $\angle FAB = \angle FAE + \angle EAC + \angle CAB = 40° + 30° + 40° = 110°$. $\boxed{\text{A}}$