# Solutions

## 1. C

$y = x^2 - 4x + 11$

Turning point: $x = \frac{-b}{2a} = \frac{-(-4)}{2(1)} = 2$

$y = (2)^2 - 4(2) + 11 = 7$

$[7, \infty)$

## 2. C

$f(3) = 5 + 3^2 = 14$

$f(4) = 14 + 16 = 30$

$f(5) = 30 + 25 = 55$

## 3. C

$m_1 = \frac{3}{4}, m_2 = \frac{-2}{3}$

$$\tan(\theta) = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \frac{\frac{3}{4} + \frac{2}{3}}{1 + \left(\frac{3}{4}\right)\left(\frac{-2}{3}\right)} = \frac{\frac{17}{12}}{\frac{1}{2}} = \frac{17}{6}$$

## 4. C

$$\ln\left|3e^{5i+2} \cdot 7e^{-2i}\right| = \ln\left(\left|21e^2\right| \cdot \left|e^{3i}\right|\right) = \ln\left(21e^2\right) = 2 + \ln 21$$

## 5. A

$y^2 + 4y + 8x + 28 = 0$ means $(y+2)^2 = -8(x+3)$. The vertex is $(-3, -2)$, and a=2, so x=-1.

## 6. A

A plane has x, y, and z coefficients equal to the components of its perpendicular vector. $x - 2y + 3z = d$

$2 + 6 - 12 = -4 \quad x - 2y + 3z = -4$

## 7. B

$e^x + e^{-x} = 2e^x - 2e^{-x}$; $3e^{-x} = e^x$; $3 = e^{2x} \qquad 2x = \ln(3) \qquad x = \frac{\ln(3)}{2}$

## 8. A

$\left(\cos^2(1°) + \cos^2(89°)\right) + \left(\cos^2(2°) + \cos^2(88°)\right) + ... + \cos^2(45°) + \cos^2(90°)$

Since $\cos x = \sin((90 - x)°) \Rightarrow \cos(89°) = \sin((90 - 89)°) = \sin(1°)$

So… $\left(\cos^2(1°) + \sin^2(1°)\right) + \left(\cos^2(2°) + \sin^2(2°)\right) + ... + \cos^2(45°) + \cos^2(90°) \Rightarrow 44(1) + \frac{1}{2} = \frac{99}{2}$

## 9. A

Factor out a -1 from the first denominator:

$\frac{-(\sin(x) + 1)}{1 - \sin(x)} + \frac{1}{1 - \sin(x)} = -1$ gives $\frac{-\sin(x)}{1 - \sin(x)} = -1$; $\frac{\sin(x)}{1 - \sin(x)} = 1$

$\sin(x)=1-\sin(x)$ $\quad 2\sin(x)=1$ $\quad \sin(x)=\frac{1}{2}$ to give $x=\frac{\pi}{6}$ and $\frac{\pi}{6}\cdot\frac{3}{4\pi}=\frac{1}{8}$

### 10. E

$${}_{12}C_4=\frac{12!}{4!8!}=495$$

### 11. C

A matrix does not have a multiplicative inverse iff the determinant is 0.

### 12. D

The sum of an infinite geometric series is the first term divided by the complement of the ratio. The first term is 2, and the common ratio is $\frac{1}{3}$, so the sum is $\frac{2}{\frac{2}{3}}=3$.

### 13.E

$$\begin{array}{llllll} & \quad\quad 1 & \quad\quad 4 & \quad 1 & \quad 0 & \\ 366_{10}=x_6 & 6^3\overline{)366} & 6^2\overline{)150} & 6^1\overline{)6} & 6^0\overline{)0} & x=1410 \\ & -216 & -144 & -6 & & \\ & 150 & 6 & 0 & & \end{array}$$

### 14. A

I'm asking for $\frac{\sin(2\theta)}{\cos(2\theta)}=\tan(2\theta)=\frac{B}{A-C}==\frac{3}{5-4}=3$

### 15. B

$$\tan(165°)=\tan((120+45)°)=\frac{\tan(120°)+\tan(45°)}{1-(\tan(120°))(\tan(45°))}=\frac{-\sqrt{3}+1}{1-(-\sqrt{3})1}=\frac{1-\sqrt{3}}{1+\sqrt{3}}=\sqrt{3}-2$$

### 16. D

The centroid is $(x,y)$, where $x$ is the average of the $x$-coordinates of the heptagon and $y$ is the average of the $y$-coordinates of the heptagon. So, $(x,y)=\left(\frac{-6-3+2+4+3+1-2}{7},\frac{2-2-3-1+4+6+7}{7}\right)=\left(-\frac{1}{7},\frac{13}{7}\right)$. The distance from this point to the line $7x-7y-70=0$ can be found using the distance from a point to line formula: $D=\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}$ where $A$, $B$, and $C$ are from a line of the form $Ax+By+C=0$ and $x_1$ and $y_1$ are the $x$-coordinate and $y$-coordinate respectively of the point you're using. So

$D=\frac{Ax_1+By_1+C}{\sqrt{A^2+B^2}}=D=\frac{7\left(-\frac{1}{7}\right)+(-7)\left(\frac{13}{7}\right)+(-70)}{\sqrt{(7)^2+(-7)^2}}=6\sqrt{2}$. Applying the formula given to us in the problem: $V=2\pi MN=2\pi\left(6\sqrt{2}\right)\left(\frac{131}{2}\right)=786\pi\sqrt{2}$.

**17. A**

$$b^a = \sin(x)$$

$$\cos^2(x) = 1 - b^{2a}$$

$$\frac{\cos^2(x)}{1+\sin(x)} = \frac{1-b^{2a}}{1+b^a} = 1 - b^a$$

**18. D**

You have two points $(x, f(x)) \to (x, x^2-8)$ and $(0,5)$. Plug those two points into the distance formula and optimize:

$$d = \sqrt{(x-0)^2 + ((x^2-8)-5)^2} = \sqrt{x^4 - 25x^2 + 169}$$

$$d' = \frac{2x^3 - 25x}{\sqrt{x^4 - 25x^2 + 169}} = 0$$

$$x = 0, \pm\frac{5\sqrt{2}}{2} \to x = \frac{5\sqrt{2}}{2}$$

$$d\left(\frac{5\sqrt{2}}{2}\right) = \sqrt{\left(\frac{5\sqrt{2}}{2}\right)^4 - 25\left(\frac{5\sqrt{2}}{2}\right)^2 + 169} = \frac{\sqrt{51}}{2}$$

**19. C**

Since 6 is even, there are 12 petals to this rose. There is an axis of symmetry that goes through the middle of each petal (and the opposite one through the origin, for 6 axes). Also, there is one in between each pair of adjacent petals. Thus, a total of 12.

**20. B**

$$(4cis(217°))(7cis(98°))\left(\frac{1}{2}cis(53°)\right) = 14cis(368°) = 14cis(8°)$$

**21. D**

Using constructive counting, there is 1 choice for the first digit, 4 for the second, 8 for the third, 7 for the fourth, and 6 for the fifth. There are thus $1(4)(8)(7)(6) = 1344$ numbers.

**22. C** 1 is constant therefore its average on any interval is itself

**23. B** $2\sin x\cos x = \sin(2x)$ So simplifying we get $\lim_{x\to 0}\frac{\sin(2x)}{2x}$ since $\lim_{x\to 0}\frac{\sin(ax)}{bx} = \frac{a}{b}$ we get $\frac{2}{2} = 1$

**24. B**

$$x^2 + 2xy + y^2 = 16 \Rightarrow x^2 + 4 + y^2 = 16 \Rightarrow x^2 + y^2 = 12 \Rightarrow x^6 + 3x^4y^2 + 3x^2y^4 + y^6 = 1728$$

$$x^6 + y^6 = 1728 - (3)(4)(12) = 1584$$

**25. B**

$$V = \frac{1}{3}\pi r^2 h \qquad V = 6\pi \qquad h = 2 \qquad \frac{dr}{dt} = 3$$

$$6\pi = \frac{1}{3}\pi r^2(2) \to r = 3$$

$$\frac{dV}{dt} = \frac{1}{3}\pi r^2\frac{dh}{dt} + \frac{1}{3}\pi 2h\frac{dr}{dt} = 0$$

$$\frac{1}{3}\pi(3)^2\frac{dh}{dt}+\frac{1}{3}\pi 2(3)(2)(3)=0\rightarrow\frac{dh}{dt}=-4$$

**26. B**

$2(2)^4-k(2)^2-4(2)+2=6 \quad 4k=20 \quad k=5$

**27. D**

$x^2=\sec^2(\pi t); \quad y^2=\tan^2(\pi t); \quad x^2-y^2=1$

**28. C**

$$\log\left(\frac{n!}{(n-1)(n-2)(...)(1)}\right)=\log(n)$$

**29. D**

$$P(B)=\frac{P(AB)}{P(A\,|\,B)}=\frac{(.4)(.6)}{.3}=.8$$

**30. D**

This is the area of a semicircle with radius 4. Thus $\frac{1}{2}\pi 4^2=8\pi$

## Tiger StateWide

1. C
2. C
3. C
4. C
5. A
6. A
7. B
8. A
9. A
10. E
11. C
12. D
13. E
14. A
15. B
16. D
17. A
18. D
19. C
20. B
21. D
22. C
23. B
24. B
25. B
26. B
27. D
28. C
29. D
30. D