Calculus TEAM SOLUTIONS – Tiger StateWide February 27, 2010

1.	A= 8	B= 1	C= -1	D= 0
2.	A= $-\frac{1}{3}$	B= $\frac{2}{3}$	C= $\frac{9}{8}$ or 1.125	D= $\frac{3}{2}$ or 1.5
3.	A= -2	B= 114	C= $\frac{\pi}{4}$	D= 4
4.	A= 8	B= 29	C= 7	D= 3
5.	A=3	B=4	C= $\frac{3}{2}$ or 1.5	D=14
6.	A= $-\frac{1}{9}$	B= $\frac{1}{36}$	C= $-2\sqrt{3}$	D= $\frac{\pi}{4}$
7.	A= $\frac{75}{2}$ or 37.5	B= $\frac{148}{9}$	C= 45	D= -1
8.	A= $-\frac{4}{3}$	B= $-\frac{\sqrt{2}}{4}$	C= 5	D= $\frac{3}{4}$ or 0.75
9.	A=56	B= 3	C= 12	D= -4
10.	A= $\frac{1}{6}$	$B=\frac{2}{15}\pi$	$C=\frac{1}{30}$	$D=\frac{\pi}{6}$
11.	A= 20π	B= 400π	C= 96π	D= 5/48
12.	A= $\sqrt{2}$	B= -6	C= -1	D= $2x$+2
13.	A= -100	B=10	C= -277.5 or -555/2	D= $-2\sqrt{2}$
14.	A=51	B= -51	C=83	D=4

1. A: Trapezoid area minus triangle area = $\frac{1}{2}(3)(5+3)-\frac{1}{2}(4)(2)=12-4=8$

B: The area is 1 only once, between g=0 and t=1.

C: $\frac{1}{9-5}\int_5^9 g(t)dt=-1;$ D: 0, since the derivative of g is shown and there is a value everywhere.

2. A: $\frac{3}{1+(3x)^2}=\frac{3}{2}$ solves to $x=\pm\frac{1}{3}$ so A= -1/3. B: $\frac{-4}{5-x}=\frac{-12}{13}$ solves to x=2/3.

C: $2x-3=\frac{-3}{4}$ solves x=9/8. D: $18x-9=8x+6$ at x=15/10 = 3/2.

3. $A=\int_{-1}^{1}\left(1-6x^2+4x^3\right)dx = x-2x^3+x^4\Big|_{-1}^{1}=(1-2+1)-(-1+2+1)=-2$

$B=\int_0^2 x(1-6x)^2\,dx = \int_0^2 (x-12x^2+36x^3)dx=\frac{x^2}{2}-4x^3+9x^4\Big|_0^2$ = (2-32+144)-(0)=114

$C=\int_{-\frac{2}{3}}^{0}\frac{6}{4+9x^2}dx = Arc\tan\left(\frac{3}{2}x\right)\Big|_{-2/3}^{0}=(0)-Arc\tan(-1)=\frac{\pi}{4}$

$\ln(D)=\frac{4}{3}\bullet\int_0^{\frac{7}{16}}\frac{1}{1-2x}dx$, $\frac{4}{3}\left(-\frac{1}{2}\right)\ln|1-2x|\Big|_0^{7/16}=-\frac{2}{3}(\ln(1/8)-0=\ln\left(\frac{1}{8}\right)^{-2/3}=\ln 4$

so D=4.

4. A: At that point, legs of the triangle are 4 and 2, so hyp= $2\sqrt{5}$. The derivative with respect to time of the Pythagorean Theorem is $\not{2}x\frac{dx}{dt}+\not{2}y\frac{dy}{dt}=\not{2}z\frac{dz}{dt}$. 4(3)+2(2)= $2\sqrt{5}$ dz/dt; $\frac{dz}{dt}=\frac{8}{\sqrt{5}}$ and A=8.

   B: The triangle has sides 7, 4 and $\sqrt{65}$; 7(3)+4(2)= $\sqrt{65}$ dz/dt; dz/dt= $\frac{29}{\sqrt{65}}$ so B=29.

   C: $\frac{dA}{dt}=\frac{1}{2}(b\frac{dh}{dt}+h\frac{db}{dt})=\frac{1}{2}(2(3)+2(4))=7$.

   D: No calculus here. $(3t+1)^2+(2t)^2=136$, $13t^2+6t-135=0$, (13t+ 45)(t-3)=0 so t=3.

5. f(0)=1 and f(2)= -67, m=-68/2= -34, so $6x^2-18x-24=-34$, $3x^2-9x+5=0$

   $x=\frac{9\pm\sqrt{21}}{6}$ so A=3.

   B: $6x^2-18x-24=0$, $6(x-4)(x+3)$ =0 and the positive value is x=4.

   C: 12x-18=0 at x=3/2.

   D: f(0)=1, f(-2)=-3 and the critical point in the interval is x= -1 so f(-1)=14. Max = 14.

6. A:, $f'=2\sqrt{x^2-1}+x(x^2-1)^{-1/2}(2x)$ =2(2)+ $\sqrt{5}$ (1/2)(2 $\sqrt{5}$)=9. Normal = -1/9.

   B: $\frac{\frac{1}{2}x^{-1/2}(1+\sqrt{x})-\frac{1}{2}x^{-1/2}(\sqrt{x})}{(1+\sqrt{x})^2}$ = $\frac{\frac{1}{2}\bullet\frac{1}{2}(3)-\frac{1}{2}\bullet\frac{1}{2}(2)}{9}=\frac{1}{36}$

   C: $h(x)=2-4\sin^2 x$ =$2(1-2\sin^2 x)=2\cos(2x)$; $h'=-4\sin(2x)=-4\bullet\frac{\sqrt{3}}{2}=-2\sqrt{3}$.

   D: $-4\sin(2x)=-4, 2x=\frac{\pi}{2}, x=\frac{\pi}{4}$

7. A: $\frac{1}{2}(3)(f(-1)+2(f(2)+f(5))+f(8))$ = 3/2 (-3+2(0+8)+12) = 75/2

   B: $\frac{1}{9-(-1)}(f(9)-f(-1))=15, K-(-3)=150, K=147$ so $\frac{1}{9-0}(147-(-1))=\frac{148}{9}$.

7. C: left = 3(-3+0+8)=15. right = 3(0+8+12)=60. Difference is 45.

   D: f' is positive and increasing at a decreasing rate so f is concave down at that point. D= -1.

Unbelievers can estimate f' for x= -0. 5 and x=0. 5 and then use these values to estimate the derivative of f' and will see a negative slope of f'.

8. The ellipse has equation $4x^2+9y^2=36$ and the circle has equation $x^2+(y-2)^2=9$

A: At x=2, y=$\frac{2\sqrt{5}}{3}$ in QI. $8x+18y\frac{dy}{dx}=0$ and A= -4/3.

B: At x=1, y=$2+2\sqrt{2}$ in QI. $2x+2(y-2)\frac{dy}{dx}=0$, dy/dx=- x/(y-2) =- 1/2$\sqrt{2}$ = -$\frac{\sqrt{2}}{4}$

C: The curve is concave down for 5 values, -2, -1, 0, 1, 2. So C=5.

D: The area under the curve from [0, 3] is 1/4 of the circle, plus 6 sq. units. Avg= $\frac{1}{3}(\frac{1}{4}\bullet 9\pi+6)=\frac{3}{4}\pi+2$ so D= 3/4.

9. A: $\int_0^4 (6x+1)dx=52$, $\int_0^4 (12x+3)dx=108$. The difference is 56.

B: $kx^2-\frac{x^3}{3}\Big|_0^k=18$, $k^3-\frac{k^3}{3}=18$, $k^3=27$, k=B=3.

C: We want the limit as x approaches 2 of the top part, which is 12.

D: To be continuous, set parts equal at x=1: 2a+b+6a=a-b+2, 7a +2b = 2.
For differentiability, set derivatives equal at x=1: 4a+b = a, 3a= -b.
So a=2 and b= -6 for a sum= -4. D= -4.

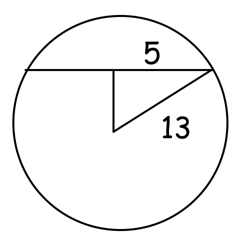
10. A: $\int_0^1 (x-x^2)dx=\frac{x^2}{2}-\frac{x^3}{3}\Big|_0^1=\frac{1}{6}$. B: $\pi\int_0^1 (x^2-x^4)dx=\frac{2}{15}\pi$

C: $\int_0^1 (x-x^2)^2dx=\int_0^1 (x^2-2x^3+x^4)dx=\frac{1}{3}-\frac{1}{2}+\frac{1}{5}=\frac{1}{30}$.

D: $\pi\int_0^1\left[\left(\sqrt{y}\right)^2-y^2\right]dy=\frac{\pi}{6}$

11. A: $\frac{dA}{dt}=2\pi r\frac{dr}{dt}$, $\frac{dA}{dt}=2\pi(5)(2)=20\pi$. B: $\frac{dV}{dt}=4\pi r^2\frac{dr}{dt}=(100\pi)4=400\pi$

C: When V=36π, r=3. $\frac{dS}{dt}=8\pi r\frac{dr}{dt}=8\pi(3)4=96\pi$



D: $x^2+y^2=169$, $2x\frac{dx}{dt}+2y\frac{dy}{dt}=0$.
2(5)(0.25)+2(12)dy/dt=0. 5/2= -24dy/dt. dy/dt= -5/48.
Answer requested was an absolute rate, so D=5/48.

12. A: $f'(x)=2x+2$, $f(g(k))=(3-k^2)^2+2(3-k^2)+1=f'(g(k))=2(3-k^2)+2$
Subtracting $2(3-k^2)$ from both sides, collecting like terms, $(3-k^2)^2=1$ and $3-k^2=\pm 1$
Solving for k leads to $k=\pm 2$ or $\pm\sqrt{2}$. Disregard the integers and negative k. $k=\sqrt{2}$

B: $g'(x)=-2x$, so $g'(2c)-12=-2(2c)-12=-4c-12$. $g'(c)=-2c$. Solving for c, $-4c-12=-2c$ and $c=-6$

C: the limit is -1. delete this

D: the symmetric difference quotient definition of the derivative, gives f'= 2x+2.

13. A: h'= f'(g(x)) g'(x) so f'(g(2))g'(2) = f'(5)(-2.5) = 40(-2.5) = -100.

B: h' = g'(f(x)) f'(x) so g'(f(6)) f'(6)=510. g'(160) 51 = 510 so g'(160)=10.

C: Using the Fundamental Theorem of Calculus, h'= $f(g(x))\bullet g'(x)$ = f(5)(-2.5) = 111(-2.5) = -277.5.

D: h' = $(g(x))^{-1/2}g'(x) = 2^{-1/2}(-0.4)$, 10g'(5)= $\frac{-4}{\sqrt{2}} = -2\sqrt{2}$

14. A: $\frac{d^2y}{dx^2}$ =12x+y' = 24+6(4)+3 = 51.

B: y-3= 27(x-2) has y-intercept y-3=-54, y= -51.

C: y-3 = -1/27 (x-2) has x-intercept 3(27)+2 = 83

D: At a critical point $\frac{dy}{dx} = 0$, so $6x^2 + y = 0, \quad y = -6x^2$. $\frac{d^2y}{dx^2}$ = $12x + \frac{dy}{dx}$ when $12x + (6x^2 + y) = 48$ and substituting to get $12x - y + y = 48$ at x=4.