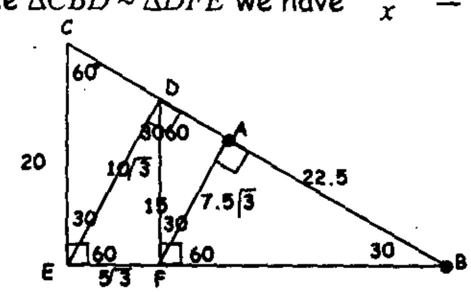
Solutions to Geometry Team Questions

1. Using the geometric mean formulas: $6^2 = (AB)(10)$ so AB=3.6 and BC=6.4. $(BD)^2 = 3.6(6.4) = 4.8$ So the sum of BC and DB is 6.4 + 4.8 = **11.2**
2. Surface area of the cube is 6(144) minus two circles on two faces $2(4\pi)$ plus the lateral area of a cylinder $(2\pi rh) = (2\pi \bullet 2 \bullet 12)$ which is $864 - 8\pi + 48\pi = 864 + 40\pi$
3. The shaded region is $16\pi - 9\pi + 4\pi - 1\pi = 10\pi$. The unshaded region is $9\pi - 4\pi + 1\pi = 6\pi$ and the ratio $\frac{6\pi}{10\pi} = \frac{3}{5}$.
4. AF is 5, by def. of the median. To find length AF we use $6^2 = x(10)$ which gives AE = 3.6, and the bisector is found with a ratio: $\frac{6}{x} = \frac{8}{10-x}$ so AD = $\frac{30}{7}$. The least of the three is 3.6 so E is farthest from C. The answer is **E**.
5. The value of x is 540-(100+110+95+150)=85. The triangle with angle A has angles of measure 30, 95 and therefore A has measure 55. The triangle at B has angles 85, 30 and B is therefore 65. The sum of the two is **120**.
6. Since a triangle inscribed in a semicircle is a right triangle, AC= 10 and $AB = 5\sqrt{3}$. The latter is x. The area of the shaded region is $\frac{1}{2}\pi(5^2) - \frac{1}{2}(5)(5\sqrt{3}) \approx 17.619$. Divide by $\pi \approx 5.608$. The product is approximately **48.6**.
7. The length of side $\overline{AB}$ is 10 so the height is $5\sqrt{3}$. Five units from vertex A is (6,0) and point C must be at $(6, -5\sqrt{3})$.
8. Using the distance formula, the side lengths are AB= 10, AC= $\sqrt{64+16} = \sqrt{80}$, and BC= $4 + 16 = \sqrt{20}$ Since the sides do satisfy the Pythagorean Theorem, ABC is a right triangle. It is not isosceles. Since $\overline{AB}$ is the longest side, angle C is the largest angle, not the smallest. So T=0+2= **2**
9. 2 X 3 = 6. The answer is **6**.
10. Let CB be (12-x) and BD=x, and AF=x and AE=(5-x). Since $\Delta CBD \sim \Delta DFE$ we have $\frac{12-x}{x} = \frac{x}{5-x}$ and $x = \frac{60}{17}$ or $3\frac{9}{17}$.
11. See diagram: 22.5



12. Since angle DCA is 60, triangle ADC is equilateral. So CD=CB=12. Dropping an altitude from C to the longest side of CDB, we get two 30-60-90 triangles. The altitude is 6 and the length of BD is $12\sqrt{3}$.
13. (BE)(ED)=(AE)(EC) so x(10-x)=(6)(4) so $10x - x^2 = 24$ and factoring $x^2 - 10x + 24 = 0$ to $(x-6)(x-4) = 0$ and x is either 6 or 4. The answer is **6**.
14. The height from B to side $\overline{AC}$ (let X be the point on $\overline{AC}$) is $5\sqrt{3}$ and the distance from X to A is 5. Consider now triangle ABX: $(5\sqrt{3})^2 + 3^2 = (BD)^2$ so BD=$84 = 2\sqrt{21}$
15. If 8 is the largest side then $6^2 + 8^2 < x$ for the triangle to be obtuse. If 8 is the largest side then $\sqrt{8^2 - 6^2} > x$ so $x < 2\sqrt{7}$. By the Triangle Inequality Theorem, $2 < x < 14$ so the intersection of the possibilities is $2 < x < 2\sqrt{7}$ *and* $10 < x < 14$ so AB = $20\sqrt{7}$