

Question #1

$$A = \int_3^1 (3x^2 - 2x + 2) dx$$

$$B = \text{the value of } f'(2) \text{ if } f(x) = x^3 - x^2 + x - 2$$

$$C = \lim_{x \rightarrow -\infty} \frac{37429x^2 - 66x^3 - 12473}{6x^3 + 47923x - 97624}$$

$$D = \text{the minimum value of } f(x) = x^2 - 2x + 6$$

Compute the value of the determinant $\begin{vmatrix} A & B \\ C & D \end{vmatrix}$.

Question #2

Which of the following statements concerning the graph of $f(x) = ax^3 + bx^2 + cx + d$ are true? Assume $a, b, c,$ and d are all rational and nonzero. (Write the letters of the correct statements on your answer sheet.)

(A) $f(x)$ has a point of inflection at $x = -\frac{b}{3a}$.

(B) The distance along the x -axis between the two extrema of $f(x)$ is $\frac{\sqrt{b^2 - 3ac}}{3a}$.

(C) The graph of $f(x)$ is tangent to the x -axis at $x = -1$ if and only if $4a - 3b + 2c - d = 0$.

(D) The leftmost of the two extrema is a local minimum if and only if $a > 0$.

Question #3

Let R denote the region of the plane bounded by the graphs of the lines $y = 2x + 3$, $y = 2$, $x = 0$, and $x = 1$.

$$A = \text{the volume of the solid formed by revolving } R \text{ about the line } y = -2$$

$$B = \text{the volume of the solid formed by revolving } R \text{ about the line } x = -2$$

Compute $A - B$.

Question #4

Let αR^3 = the volume of the largest cylinder that can be inscribed in a sphere of radius R . Find α .

Question #5

If $f(x) = (3x^4 + 7)^3 (2 - 5x)^6 \cos^5\left(x - \frac{\pi}{6}\right)$, then what is the value of $f'(0)$?

Question #6

Let βR^3 = the volume of the largest cylinder that can be inscribed in the largest cone that can be inscribed in a sphere of radius R . Find β .

Question #7

$$\text{Compute } \int_{-5}^2 |x^3 - 2x^2 - 9x + 18| dx.$$

Question #8

$$\text{Let } f(x) = \sqrt{36 - x^2}.$$

$$M = \int_{-6}^6 f(x) dx$$

$$A = \text{the area of the region bounded by the } y\text{-axis, the graph } f(x), \text{ and the line } y = x \tan \theta \text{ for } 0 < \theta < \frac{\pi}{2}$$

$$R = \text{the arc length of the curve } f(x) \text{ from } (-3\sqrt{3}, 3) \text{ to } (3\sqrt{2}, 3\sqrt{2}).$$

$$K = \text{the slope of the curve } f(x) \text{ at the point where } f(x) \text{ intersects the line } y = bx$$

Compute the product $MARK$.

Question #9

Mrs. Singleton and Mr. Bantz are chatting about their favorite functions. Mrs. Singleton tells Mr. Bantz that her favorite function is a cubic polynomial function with rational coefficients. “Go on,” he says, intrigued. Mrs. Singleton continues, “well, it has a point of inflection at $(2, -6)$, but my favorite part is the local maximum at $(-1, 48)$.” At this point, Mr. Bantz doesn’t need any more clues. “Oh, *that* one,” he says, “My wife loves that one.” Find the sum of the coefficients of Mrs. Singleton’s favorite function.

Question #10

Find the sum of the positive numbers A and B which satisfy the following equalities.

$$\int_0^A w e^{w^2} dw = 9$$

$$\int_1^B y \ln y dy = \frac{1}{4}(e^2 + 1)$$

Question #11

Let $f(x) = x^2 - 2x - 8$.

A = the value of x guaranteed by Rolle’s Theorem on $[-2, 4]$

B = the value of x guaranteed by the Mean Value Theorem for Derivatives on $[6, 12]$

C = the average value of $f(x)$ on $[-6, 9]$

D = the minimum value of $f(x)$

Compute $A + B + C + D$.

Question #12

Each of the following statements concerning the function $f(x) = 3x^4 - 10x^3 - 27x^2 + 108x - 74$ has a value, indicated to the left of each statement. Find the sum of the values of the correct statements.

(-4) $f(x)$ has an x -intercept at $x = 1$.

(1) $f(x)$ has a point of inflection at $(2, 2)$.

(0) $f(x)$ has a local minimum at $(-2, -270)$.

(5) $f(x)$ has a y -intercept at $y = -74$.

(-2) $f(5) = 376$

Question #13

Compute $\int_{\frac{3}{2}}^{\frac{9}{4}} \frac{dx}{\sqrt{3x - x^2}}$.

Question #14

$$A = \lim_{h \rightarrow 0} \frac{f(1) - f(1-h)}{h} \text{ for } f(x) = \ln x$$

$$B = \lim_{h \rightarrow 0} \frac{g\left(\frac{\pi}{6} + h\right) - g\left(\frac{\pi}{6}\right)}{h} \text{ for } g(x) = \cot x$$

$$C = \lim_{y \rightarrow 3} \frac{h(y) - h(3)}{y - 3} \text{ for } h(x) = x^3 + 2x^2 + 3x + 4$$

Find $A + B + C$.

Question #15

Compute $\int_{\frac{\pi}{6}}^{\frac{5\pi}{4}} e^{4x} \cos 5x dx$.