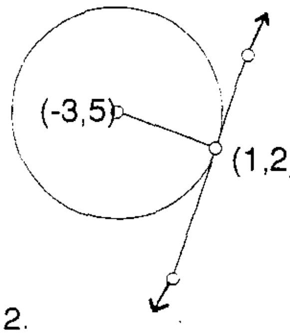
Geometry Team Questions
FAMAT Regional-February
Answer Key

1. $4x - 3y = -2$
2. $\frac{3\sqrt{2}}{4}$
3. $\frac{5}{6}$ and $\frac{5}{12}$
4. $\frac{29\pi}{2}$ or 14.5π
5. $\frac{45}{7}$ or $6\frac{3}{7}$
6. 52
7. $2\sqrt{73}$
8. 66
9. $2\pi\sqrt{2}$
10. $350 + 72\sqrt{3}$
11. $\frac{40\pi}{3} - 16\sqrt{3}$ or $\frac{40\pi - 48\sqrt{3}}{3}$
12. 30
13. 14
14. 90.2
15. 42π

FAMAT Regional Geometry Team Solutions February

1.



$x^2+y^2+6x-10y+9=0$

$(x+3)^2+(y-5)^2=25$

$\therefore$ center $=(-3,5)$

$\therefore$ slope of radius $=\frac{-3}{4}$; $\therefore$ slope of tangent $=\frac{4}{3}$

$\therefore$ equation of tangent line is: $y-2=\frac{4}{3}(x-1)$

$\therefore$ in standard form, is: $\boxed{4x-3y=-2}$

2.

$$\frac{A_{\text{hexagon}}}{A_{\text{square}}}=\frac{3\sqrt{3}}{4}=\frac{3x^2\sqrt{3}}{4x^2}$$

$$A_{\text{hexagon}}=6\left(\frac{s^2\sqrt{3}}{4}\right)=\frac{3}{2}s^2\sqrt{3}=3x^2\sqrt{3}\ \therefore s=x\sqrt{2}\ \therefore P=6x\sqrt{2}$$

$$A_{\text{square}}=s^2=4x^2\quad \therefore s=2x\quad \therefore P=8x$$

$$\therefore \frac{P_{\text{hexagon}}}{P_{\text{square}}}=\frac{6x\sqrt{2}}{8x}=\boxed{\frac{3\sqrt{2}}{4}}$$

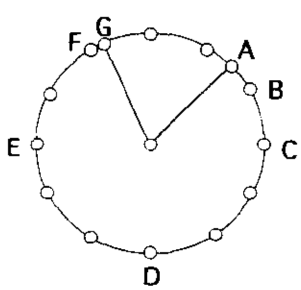
3.

x = length of the longer part and $5-x$ = length of the shorter part.

$\therefore x^2=4(5-x)^2 \Rightarrow 3x^2-40x+100=0 \Rightarrow$ ~~$x=10$~~ or $x=\frac{10}{3}$.

$\therefore$ longer part is $\frac{10}{3}$ and shorter part is $\frac{5}{3}$ $\therefore$ sides are $\boxed{\frac{5}{6} \text{ and } \frac{5}{12}}$

4.

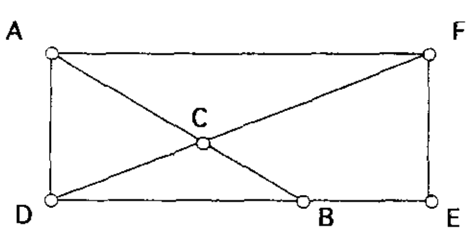


Let all points except A and G represent the hours on a clock.
Point A represents the tip of the hour hand at 1:30 PM,
and Point G represents the tip of the hour hand at 11:10 PM.

$m\widehat{AB}=\frac{30}{60}(30°)=15°$; $m\widehat{BC}=30°$; $m\widehat{CD}=m\widehat{DE}=90°$; $m\widehat{EF}=60°$

$m\widehat{FG}=\frac{10}{60}(30°)=5°\ \therefore m\widehat{ADG}=290°\ \therefore$ arc length $=\frac{290}{360}(2\pi)(9)=\boxed{\frac{29\pi}{2}}$

5.



Since $\overline{AB}$ bisects $\angle DAF$, ΔADB is a 45° - 45° - 90° $\therefore AD=9$.

$AF=12$ and $\therefore DF=15$. Let $CD=x$. In ΔADF, $\frac{AD}{AF}=\frac{CD}{CF}$

$\Rightarrow \frac{9}{12}=\frac{x}{15}\Rightarrow 135-9x=12x\Rightarrow x=CD=\frac{135}{21}=\boxed{6\frac{3}{7}}$

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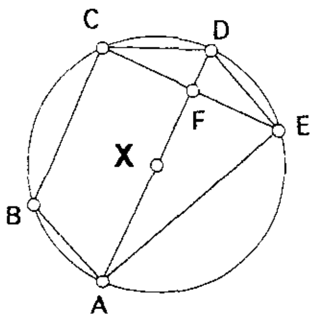
6.



$(AF)(FD) = (CF)(FE) \Rightarrow 8(2) = x^2 \Rightarrow x = 4 = CF = FE$

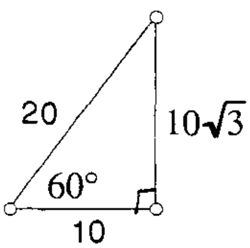
$\therefore A_{\text{trapezoidABCD}} = \frac{4}{2}(6+10) = 2(16) = 32$

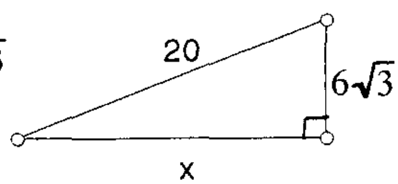
and $A_{\Delta AED} = \frac{1}{2}(10)(4) = 20$

$\therefore A_{\text{pentagonABCDE}} = 32 = 20 = \boxed{52}$

7.

Original   After 8 minutes: $8\left(\frac{\sqrt{3}}{2}\right) = 4\sqrt{3}$ down



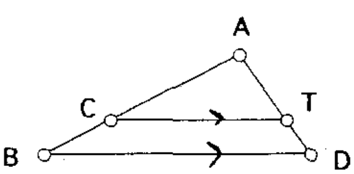


$x^2 + \left(6\sqrt{3}\right)^2 = 20^2$

$x^2 = 292$

$x = \boxed{2\sqrt{73}}$

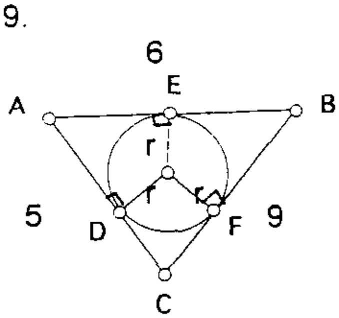
8.



$\frac{AC}{7} = \frac{12}{6}$ $\therefore AC = 14$ and $\frac{18}{BD} = \frac{12}{18}$ $\therefore BD = 27$

$\therefore$ Perimeter of $\Delta ABD = 21 + 18 + 27 = \boxed{66}$

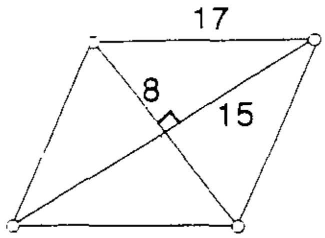
9.



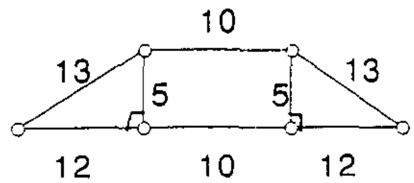
$A_{\Delta} = \frac{1}{2} r p_{\Delta}$. Using Heron's Formula, $A_{\Delta} = \sqrt{10(10-5)(10-6)(10-9)}$

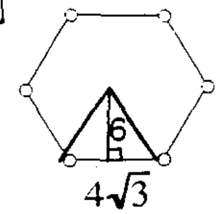
$= 10\sqrt{2}$ $\therefore 10\sqrt{2} = \frac{1}{2} r(20)$ $\therefore r = \sqrt{2}$ $\therefore C_{\text{circle}} = 2\pi(\sqrt{2}) = \boxed{2\pi\sqrt{2}}$

10.



$A + B + C = 240 + 110 + 72\sqrt{3} = \boxed{350 + 72\sqrt{3}}$



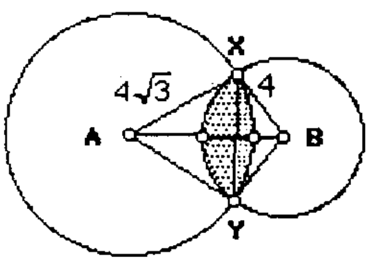


$$A = \frac{1}{2}ap$$
$$= \frac{1}{2}(30)(16)$$
$$= 240$$

$$A = \frac{h}{2}(b_1 + b_2)$$
$$= \frac{5}{2}(10 + 34)$$
$$= 110$$

$$A = \frac{1}{2}ap$$
$$= \frac{1}{2}(6)(24\sqrt{3})$$
$$= 72\sqrt{3}$$

11.



Area of Segment cut off by larger circle $= \frac{60}{360}(\pi)(4\sqrt{3})^2 - \frac{(4\sqrt{3})^2\sqrt{3}}{4}$

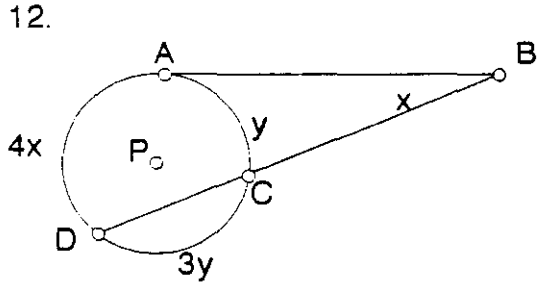
$$= \frac{1}{6}(48\pi) - \frac{48\sqrt{3}}{4} = 8\pi - 12\sqrt{3}$$

Area of Segment cut off by smaller circle $= \frac{120}{360}(\pi)(4)^2 - \frac{1}{2}(4\sqrt{3})(2)$

$$= \frac{1}{3}(16\pi) - 4\sqrt{3} = \frac{16\pi}{3} - 4\sqrt{3}$$

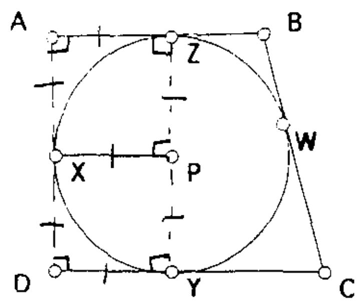
$\therefore$ Area of intersection of circles $= \boxed{\frac{40\pi}{3} - 16\sqrt{3} \text{ or } \frac{40\pi - 48\sqrt{3}}{3}}$

12.



$$4x + 4y = 360 \Rightarrow x + y = 90$$
$$x = \frac{1}{2}(4x - y) \Rightarrow y = 2x$$
$$\therefore x + 2x = 90 \Rightarrow x = \boxed{30}$$

13.



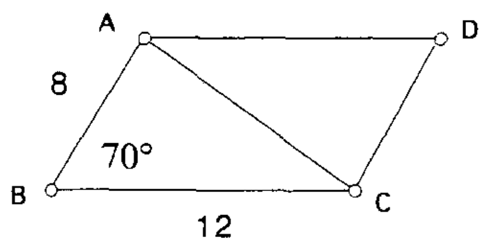
$PX = PZ = AZ = AX = DY = XD = PY = 6$

Since $AB = 10$, $\therefore ZB = 4 = BW$

Since $CD = 16$, $\therefore YC = 10 = CW$

$\therefore BC = \boxed{14}$

14.



$$A_{\Delta ABC} = \frac{1}{2}(8)(12)(\sin 70°) = 45.1052458.....$$

$$\therefore A_{parallelogram ABCD} = 2(45.1052458) \approx \boxed{90.2}$$

15.

$r = 3$

$h = 8$

$$V_{hemisphere} = \frac{1}{2}\left(\frac{4}{3}\pi(3)^3\right) = 18\pi$$

$$V_{cone} = \frac{1}{3}(\pi)(3)^2(8) = 24\pi$$

$$\therefore V_{ice\ cream} = 18\pi + 24\pi = \boxed{42\pi}$$