Solutions 1997 Alpha Team

1. A = radius of circ. circle $= \frac{A \cdot B \cdot C}{4 \text{Area}} = \frac{7 \cdot 7 \cdot 7}{4(\frac{1}{2} \cdot 7 \cdot \frac{7\sqrt{3}}{2})} = \frac{343}{49\sqrt{3}} = \frac{7\sqrt{3}}{3}$

B = radius of insc. circle $= \frac{\text{Area}}{\text{semiperimeter}} = \frac{\frac{49\sqrt{3}}{4}}{\frac{21}{2}} = \frac{7\sqrt{3}}{6}$ $A+B = \frac{7\sqrt{3}}{2}$

2. $7\sqrt{(x-2)^2+(y-2)^2} = \sqrt{(x-5)^2+(y-4)^2} = 4x^2-16x+16+4y^2-16y+16 = x^2-10x+25+y^2-8y$

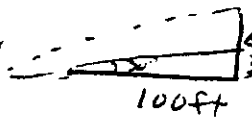
$= 3x^2-6x+3y^2-8y-9=0$

3. $A = 3(6^2)+4(6)+5 = 137$ $B = 1(2^5)+1(2^3)+2+1 = 43$

$C = 3(5^3)+2(5^2)+5+4 = 434$ $A+B+C = 614$ $614 = 15357$

4. $P = \begin{vmatrix} 1 & 2 & 4 \\ 3 & 1 & 1 \\ i & j & k \end{vmatrix} = -2i+11j-5k$ $Q = \begin{vmatrix} 1 & 2 & 4 \\ 2 & 3 & 1 \\ i & j & k \end{vmatrix} = -10i+7j-k$ $P \cdot Q = 20+77+$

$= 102$

5. 

Law of similar Δs. $\frac{Y_3 X}{X} = \frac{100}{Y}$ $Y = 300$

6. $Q = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 6 \\ 3 & 1 & 1 \end{bmatrix}$ $\det(Q) = 20 = A$ $\det(2Q) = 40 = B$ $20(6)+40(\frac{1}{20}) = 13$

$\det(Q^{-1}) = \frac{1}{20} = C$ # of 3×3 perm. mat. $= 6 = D$

7. Distance between plane #1 and origin, $\frac{3(0)+4(0)+5(0)-10}{\sqrt{3^2+4^2+5^2}} = \frac{-10}{\sqrt{50}} = -\sqrt{2}$

Distance between plane #2 and origin $= 4\sqrt{3}$ Distance $= 5\sqrt{2}$

8. A, $y = (4-x^2)^{1/2}$ $y^2+x^2=4$ $A = 4\pi$ $B = x^2-6x+9+y^2+8x+16 = 25$ $B = 25\pi$

$C = 3(x^2+6x+9)+5(y^2+2y+1) = 15$ $\frac{(x+3)^2}{5}+\frac{(y+1)^2}{5}=1$ $C = \sqrt{15}\pi$ $A+B+C = \pi(29+\sqrt{15})$

9. $A = 6^{550}$ $6 \equiv -1 \bmod 7$ $(-1)^{550} = 1$ $A = 1$ $11 \equiv -1 \bmod 6$ $11^{721} \equiv (-1)^{721}$ $B = 5$

$13 \equiv 3 \bmod 5$ $(13)^4 \equiv 1 \bmod 5$ $1^{36}(13^3) \equiv 2 \bmod 5$ $C = 2$ $5+10 = 15$

10. $A = \frac{n(n+1)}{2} = \frac{100(101)}{2} = 5050$ $B = n(n+1) = 100(101) = 10{,}100$ $C = n^2 = 100^2 = 10{,}000$

$D = \frac{n(n+1)(2n+1)}{6} = \frac{50(51)(101)}{6} = 42{,}925$ $A+B+C-D = -17{,}775$

11. $\frac{1}{8}(\frac{1}{4})+\frac{1}{8}(\frac{1}{6})+\frac{1}{8}(\frac{1}{12}) = \frac{1}{72}+\frac{1}{48}+\frac{1}{96}$ $\frac{13}{288}$

12. $(10, 60°) = (5\sqrt{3}, 5)$ $(8, 150°) = (4, -4\sqrt{3})$ $(6, 270°) = (-6, 0)$ Sum of coord. $(5\sqrt{3}-2, 5-4-$

Magnitude $= (5\sqrt{3}-2)^2+(5-4\sqrt{3})^2 = 152-60\sqrt{3}$

13. Make vectors $(3,1,4)-(1,0,1) = (2,1,3)$ and $(6,2,3)-(1,0,1) = (5,2,2)$

Area $= \left[\begin{vmatrix} 2 & 1 & 3 \\ 5 & 2 & 2 \\ i & j & k \end{vmatrix}\right] \div 2 = [-4i+11j+k] \div 2 = \sqrt{16+121+1}/2 = \sqrt{138}/2$

14. $A = -32$ $B = i$ $C = -117+44i$ $D = 32$ $A+B+C-D = 53-43i$ $m+n = 10$

15. $A = \frac{6!}{2!} = 360$ $B = \frac{5!}{2!} = 60$ $A+B = 420$