CALCULUS TEAM QUESTIONS

VERO BEACH FAMAT INVITATIONAL MARCH 25,1995

All answers are to be exact unless otherwise specified

CALCULUS TEAM QUESTION 1.

Find the slope of the normal line to the lemniscate $8(x^2 + y^2)^2 = 100(x^2 - y^2)$ at $(3,1)$.

CALCULUS TEAM QUESTION 2.

$$A = \lim_{x \to -\infty} \frac{\sqrt{x^2 + 2}}{3x - 6} \qquad B = \lim_{x \to 0} \frac{\tan x + \sin x}{x \cos x}$$

$$C = \lim_{x \to 0} \frac{\tan^{-1}x - x}{8x^3} \qquad D = \lim_{x \to \infty} [\ln(x + 1) - \ln(x - 1)]$$

Find ABC-D.

CALCULUS TEAM QUESTION 3.

Find the volume of the solid generated by revolving the ellipse $(x - 4)^2 + 2y^2 = 16$ about the y-axis.

CALCULUS TEAM QUESTION 4.

The rate of decay of radium is proportional to the amount present at any time. If 60 mg of radium are present now and its half-life is 1690 years, how much radium to the nearest tenth of a gram will be present 100 years from now?

CALCULUS TEAM QUESTION 5.

Given $f(x) = 3x^2 - 15x$ and....

A = the value of **x** which satisfies the conclusion of Rolle's Theorem on $[0,5]$

B = the value of **x** satisfying the Mean Value Theorem on $[0,6]$

C = the **average value** of $f(x)$ on $[-1,0]$

Find $(A - C)/B$.

CALCULUS TEAM QUESTION 6.

The area of an equilateral triangle is decreasing at a rate of $4cm^2/min$. Find the rate at which the length of a side is changing when the area of the triangle is $200cm^2$. Round your answer correctly to 3 decimal places.

CALCULUS TEAM QUESTION 7.

$A(x) = \sinh x$ $\qquad C(t) = \int_2^{x^2} \sqrt{1 - t^2}\, dt$

$B(x) = 4^{\sin x}$ $\qquad D(x) = \cos^{-1}(2x)$

Find $A'(0) - B'(\pi/6) + C''(0) - D'(1/4)$.

CALCULUS TEAM QUESTION 8.

Find $\lim_{x \to \pi/2^-} (\tan x)^{\cos x}$.

CALCULUS TEAM QUESTION 9.

Find the length of the curve $y = x^2/4 - \ln \sqrt{x}$, $1 \leq x \leq 2$.

CALCULUS TEAM QUESTION 10.

Evaluate $\int e^x \cos x\, dx$.

CALCULUS TEAM QUESTION 11.

Determine the equation of the tangent line to $y = \ln(\log x)$ at $(10,0)$.

CALCULUS TEAM QUESTION 12.

If $x = t^2 - 1$ and $y = t^4 - t^3$, then when $t = 1$, find d^2y/dx^2.

CALCULUS TEAM QUESTION 13.

Find the area of the region bounded by the curve $y = 2xe^{-.5x}$, the x-axis, and the line $x = 4$.

CALCULUS TEAM QUESTION 14.

Evaluate $\int_0^2 \frac{x^2\, dx}{\sqrt{16 - x^2}}$.

CALCULUS TEAM QUESTION 15.

Evaluate: $\lim_{x \to \infty} \sqrt{x^2 + 2x} - \sqrt{x^2 - 2x}$.