GRENELEFE 1989 — FUNCTIONS

SOLUTIONS

1. B

$f(x+2) = (x+3)^2 + 3(x+3) - 7$

$= x^2 + 6x + 9 + 3x + 9 - 7$

$= x^2 + 9x + 11$

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2. D

$x^2 - 2x - 35 = (x-1)^2 - 36$

$\therefore y \geq -36$

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3. B

$g(f(x)) = 2x + 1$

$2x + 1 = 1$

$x = 0$

$2x + 1 = 3$

$x = 1$

$2x + 1 = 5$

$x = 2$

---

4. C

$x^2 - 7x + 6 = (x-6)(x-1)$

$2^{a+2} = 6 \quad \text{or} \quad 2^{a+2} = 1$

$2^a = \frac{3}{2} \qquad a = -2$

$a \log_2 2 = \log_2 3 - \log_2 2$

$a = \log_2 3 - 1$

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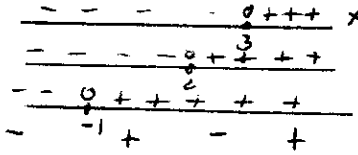
5. D

$x^3 - 4x^2 + x + 6 =$

$(x-3)(x-2)(x+1)$



< 0 for $x < -1$ or $2 < x < 3$

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6. D

$|x+5| - |x-3|$

for $x \leq -5$

$-x - 5 - (-x+3) = -8$

for $-5 < x \leq 3$

$x + 5 - (-x+3)$

$2x + 2 \Rightarrow -8 < y < 8$

for $x > 3$

$x + 5 - (x-3) = 8$

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7. B

$f(1) = 1$

$f(2) = \frac{1}{4}$

$f(3) = \frac{3}{2}$

$f(4) = \frac{13}{4}$

$f(5) = 8$

$f(6) = \frac{77}{4}$

$f(7) = \frac{77}{2} + 8 = \frac{93}{2}$

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8. B

$f(f(f(\tfrac{1}{2}))) = f(f(\tfrac{1+4}{1+2}))$

$= f(f(\tfrac{5}{3}))$

$= f(\tfrac{5+6}{8}) = f(\tfrac{11}{8})$

$= \frac{11+16}{19} = \frac{27}{19}$

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9. A

II + III only

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10. C

$\frac{24-6x}{x^2-2x-8} = \frac{6(4-x)}{(x-4)(x+2)}$

$= \frac{-6}{x+2}$ for $x \neq 4$

For integral solutions $x =$

$-8, -5, -4, -3, -1, 0, 1,$

11. $(3^x + 3^{-x})^2 - (3^x - 3^{-x})^2 =$

C $3^{2x} + 2 + 3^{-2x} - 3^{2x} + 2 - 3^{-2x} =$

4

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12. $\sqrt{x + 2\sqrt{x} + 1} = |\sqrt{x} + 1|$

D $\sqrt{x - 2\sqrt{x} - 1} = |\sqrt{x} - 1|$

for $x \geq 1$

$\sqrt{x} + 1 - \sqrt{x} + 1 = 2$

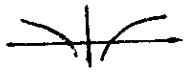
for $0 \leq x < 1$

$\sqrt{x} + 1 + 2\sqrt{x} - 1 = 2\sqrt{x}$

$\therefore$ Range $[0, 2]$

13

E

$g(x)$ · 

$|g(x)|$ = 

$f|g(x)|$ defined for $x \neq \pm 1$, $x \neq 0$

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14. $f \circ g = \{(7,3), (3,3), (5,3)\}$

A.

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15. $\frac{3}{5x+2} = \frac{15}{x+2}$

A. $3x + 6 = 75x + 30$

$-24 = 72x$

$-\frac{1}{3} = x$

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16. $f(24) = \log 24$

A. $= \log 2^3 + \log 3$

$= 3\log 2 + \log 3$

$= 1.05 + .55$

$= 1.6$

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17. $\frac{(x+h)^2 + 3(x+h) + 2 - [x^2 + 3x + 2]}{h} =$

D

$\frac{x^2 + 2xh + h^2 + 3x + 3h + 2 - x^2 - 3x - 2}{h} =$

$2x + 3 + h$

18.

D

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19. $f(\log_2 a) = 2^{\log_2 a - 2}$

D. $= 2^{\log_2 a} \cdot \frac{1}{4}$

$= \frac{a}{4}$

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20. $[f+g](4) = 29 + 19 = 48$

C $\left[\frac{f}{g}\right](-1) = \frac{-1}{4}$

$[f \circ g](-1) = f(4) = 29$

$[g \circ f](3) = g(15) = \underline{52}$

128.75

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21. $y = 3\log x + 7$

A $x = 3\log y + 7$

$x - 7 = 3\log y$

$\frac{x-7}{3} = \log y$

$10^{\frac{x-7}{3}} = y$

22. $d(d(d(k)))=$

D. $d(d(k+3))=$

$d(2k+6)=$

$4k+12$

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23. $f(x+2)=\frac{x+4}{x+2}$

A.

$xf(x)-x=2$

$x=\frac{2}{f(x)-1}$

$$\frac{\frac{2}{f(x)-1}+4}{\frac{2}{f(x)-1}+2}=\frac{2+4f(x)-4}{2+2f(x)-2}$$

$$=\frac{2f(x)-1}{f(x)}$$

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24 $\binom{n}{0}+\binom{n}{1}\left(\frac{1}{n}\right)+\binom{n}{2}\left(\frac{1}{n}\right)^2+\binom{n}{3}\left(\frac{1}{n}\right)^3=$

B $1+1+\frac{n!}{2!(n-2)!}\cdot\frac{1}{n^2}+\frac{n!}{3!(n-3)!}\cdot\frac{1}{n^3}=$

$$2+\frac{n-1}{2n}+\frac{(n-1)(n-2)}{6n^2}=$$

$$\frac{12n^2+3n^2-3n+n^2-3n+2}{6n^2}=$$

$$\frac{16n^2-6n+2}{6n^2}=\frac{8n^2-3n+1}{3n^2}$$

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25. $\log_2(x+1)<4$

D $0<x+1<16$

$-1<x<15$

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26. $-1+a=-3$

B. $a=-2$

$27=9+b$

$18=b$

$(-2,18)$

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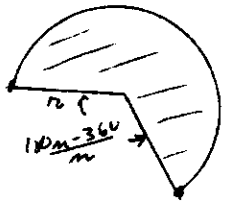
27. $f_5\circ f_6=\frac{1}{1-\frac{x-1}{x}}=$

A. $\frac{x}{x-x+1}=x=f_1$

28.

A.



$$A=\pi n^2-\left[\frac{\frac{180n-360}{n}}{360}\right]\pi n^2$$

$$\pi n^2\left[1-\frac{n-2}{2n}\right]=\pi n^2\left[\frac{n+2}{2n}\right]$$

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29. ax^2+bx+c

C $a+b+c=3$ (1)

$4a+2b+c=-1$ (2)

$9a+3b+c=5$ (3)

$3a+b=-4$

$5a+b=6$

$2a\ a=10$

$a=5,\ b=-19,\ c=17$

$P(4)=16\cdot5-19\cdot4+17=21$

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30. Permutation of 4 things where 2 are alike

C

6, 6, 7, 8
6, 7, 7, 8
6, 7, 8, 8

3: $\frac{4!}{2!}=3\cdot4\cdot3$

$=36$