Theta Bowl
1989
Solutions

1. $\left(2^3\right)^{x^2+3x+10} = \left(2^2\right)^{x^2-x}$

$\therefore\ 3x^2+9x+30 = 2x^2-2x$

$x^2+11x+30=0$ $\Longrightarrow$ sum of the roots $= \boxed{-11}$

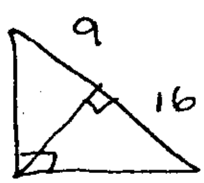
2. $\log_{10}\sqrt{a} = \frac{2}{5} \Rightarrow \log_{10} a = \frac{4}{5}$

$\log_{10} b^4 = 20 \Rightarrow \log_{10} b = 5$

$\therefore \log_{10} ab + \log_{10} a^5 = \log_{10} a + \log_{10} b + 5\log_{10} a$

$= \frac{4}{5} + 5 + 5\left(\frac{4}{5}\right)$

$= \frac{4}{5} + 9 = \boxed{\frac{49}{5}}$

3. 

$\text{Area} = \frac{r+s}{2}\sqrt{rs}$

where r, s are segments of hypotenuse

$\therefore A = \frac{9+16}{2} \cdot \sqrt{9 \cdot 16}$

$= \frac{25}{2} \cdot 12$

$= \boxed{150}$

Theta Bowl 1989 - solutions

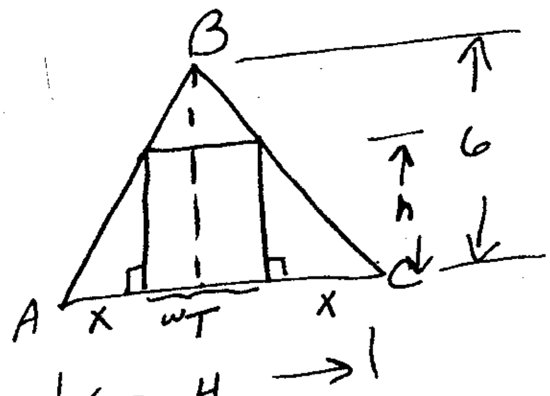
4. $4x - 8 = 0 \qquad 4x = 8 \qquad x = 2$

$y^2 - 2y - 8 = 0 \Rightarrow (y-4)(y+2) = 0 \qquad y = 4, -2$

$5z + 6 + z^2 = 0 \qquad (z+3)(z+2) = 0 \qquad z = -3, -2$

$\therefore \boxed{(2,4,-3);\ (2,4,-2);\ (2,-2,-3);\ (2,-2,-2)}$

5.



$h =$ ht of rectangle
$w =$ base of rectangle
$x =$ base of small rt Δ.

$\therefore\ 2x = 4 - w$
$w = 4 - 2x$

By similarity: $\frac{x}{h} = \frac{2}{6}$

$x = \frac{1}{3}h$

Area rectangle $= wh$

$5 = (4 - 2x)(h)$

$5 = (4 - 2(\tfrac{1}{3}h))(h)$

$5 = 4h - \tfrac{2}{3}h^2$

$2h^2 - 12h + 15 = 0$

$\therefore\ h = \frac{12 \pm \sqrt{144 - 120}}{4} = \boxed{3 \pm \tfrac{1}{2}\sqrt{6}}$

6. The area of the n-gon $=$ (Semi-perimeter) $\cdot$ (radius of circle)

$= (32)(32)$

$= \boxed{1024}$

$A_O = 1024\pi = \pi r^2$

$1024 = r^2$

$32 = r$

Theta Bowl - 1984
SOLUTIONS

7. $$-\begin{vmatrix} 3x^2 & 12x \\ 15x & 41 \end{vmatrix} + \begin{vmatrix} x^2 & 12x \\ 5x & 61 \end{vmatrix} - 7x^6 \begin{vmatrix} x^2 & 3x^2 \\ 5x & 15x \end{vmatrix} = -8$$

$$-(183x^2 - 180x^2) + (61x^2 - 60x^2) - 7x^6(15x^3 - 15x^3) = -8$$

$$-3x^2 + x^2 = -8$$

$$-2x^2 = -8$$

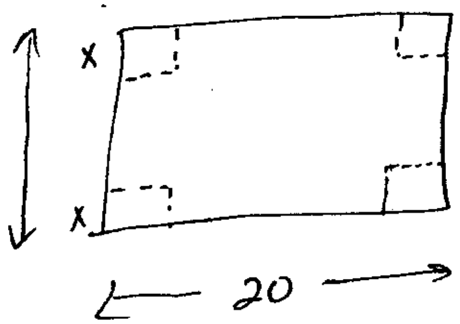
$$x^2 = 4 \Rightarrow \boxed{x = \pm 2}$$

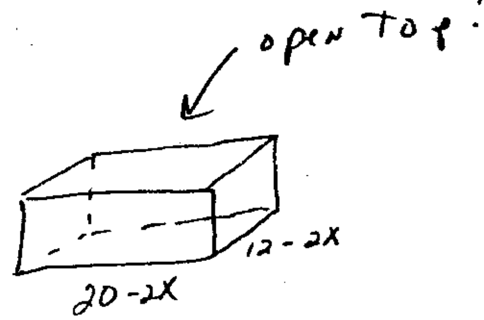
8. ${}_4C_3$ ways to choose vowels

${}_4C_2$ ways to choose consonants

$5!$ ways to permute chosen letters.

$$\therefore \text{total} = \frac{4!}{3!\,1!} \cdot \frac{4!}{2!\,2!} \cdot 5! = \boxed{2880}$$

9. 

$\Rightarrow$ 

$$A = (20-2x)(12-2x) = 48$$

$$4x^2 - 64x + 240 = 48$$

$$\therefore 4x^2 - 64x + 192 = 0$$

$$x^2 - 16x + 48 = 0$$

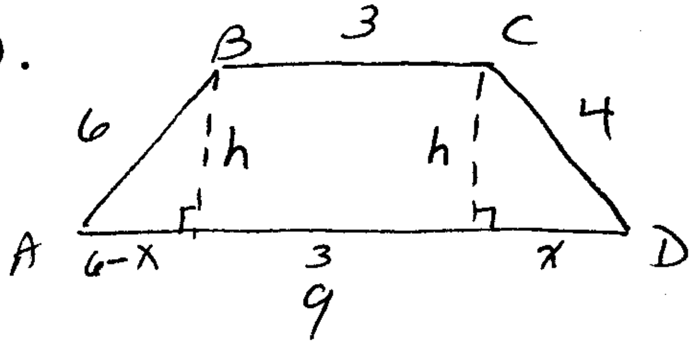
$$(x-12)(x-4) = 0 \Rightarrow x \neq 12,\ x = 4$$

$\therefore$ 

$$\text{vol} = 4^3 = \boxed{64}$$

Theta Bowl - 1989
Solutions

10.



$$\Rightarrow \quad \begin{aligned} h^2+x^2 &= 16 \\ h^2+(6-x)^2 &= 36 \end{aligned}$$

$$(6-x)^2 - x^2 = 20$$
$$36-12x+x^2-x^2 = 20$$
$$36-12x = 20$$
$$-12x = -16$$
$$x = \frac{4}{3}$$

$$\Leftarrow \quad h^2 + \frac{16}{9} = 16$$
$$h^2 = 16 - \frac{16}{9}$$
$$h^2 = \frac{144-16}{9}$$
$$h^2 = \frac{128}{9} \quad \Longrightarrow \quad \boxed{h = \frac{8\sqrt{2}}{3}}$$

11.

$$\frac{9}{a^2} - \frac{15}{b^2} = 1 \quad \Longrightarrow \quad \frac{9}{a^2} - \frac{15}{16-a^2} = 1$$
$$b^2 = 16-a^2$$

$$9(16-a^2) - 15a^2 = a^2(16-a^2)$$
$$144 - 9a^2 - 15a^2 = 16a^2 - a^4$$
$$a^4 - 40a^2 + 144 = 0$$
$$(a^2-36)(a^2-4) = 0$$

ext. $\rightarrow$ $a^2 = 36$, $a = 6$ $\qquad$ $a^2 = 4$, $a = 2$

$$b^2 = 16-4$$
$$b^2 = 12$$

$$\therefore \quad \boxed{\frac{y^2}{4} - \frac{x^2}{12} = 1}$$

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12. $\log_4(\log_5 25) = \log_3 x$

$\log_4 2 = \log_3 x$

$4^{\log_3 x} = 2^1$

$(2^2)^{\log_3 x} = 2^1 \Rightarrow 2\log_3 x = 1$

$\log_3 x = \frac{1}{2}$

$\therefore \boxed{x = 3^{1/2} = \sqrt{3}}$

13. $\boxed{2^{10} = 1024}$

14. 4^{th} term $\Rightarrow \binom{7}{3} a^4 (-2b^2)^3 = \frac{7\cdot6\cdot5}{3\cdot2\cdot1}\cdot a^4\cdot(-8)\cdot b^6$

$= \boxed{-280\, a^4 b^6}$

15. $x + \sqrt{x+5} = 7$

$\sqrt{x+5} = 7 - x$

$x + 5 = 49 - 14x + x^2$

$0 = x^2 - 15x + 44$

$0 = (x-11)(x-4)$

$x = 11, 4$

CK: $11 + \sqrt{16} \stackrel{?}{=} 7$

$11 + 4 \neq 7$

$4 + \sqrt{4+5} = 7$

$4 + 3 = 7$

$\boxed{x = 4}$