The acronym NOTA denotes that "none of these answers" are correct. DNE stands for Does Not Exist. The domain and range of functions are assumed to be the real numbers or the appropriate subset of the real numbers, unless otherwise specified. In any problem involving complex numbers, $i = \sqrt{-1}$. Have fun!
On the last page are some common values of the sine function to save tedious calculation.

1. Let $k > 0$. A circle has circumference k cm and area k cm^2. What is its radius, in cm?
A. 0.5 B. 1 C. 2 D. 4 E. NOTA

2. The *modulus* of a complex number is its absolute value. What value of x gives the least possible modulus for the complex number $z = (x + 2) + (4 - x)i$?
A. -2 B. 1 C. 2 D. 4 E. NOTA

3. A fourth-degree polynomial $P(x)$ with real coefficients has leading coefficient 1 and roots $2 + 3i$ and $4 - 3i$. What is $P(1)$?
A. 180 B. 234 C. 325 D. 442 E. NOTA

4. A 4x4 square makes one-half of a rotation about one of its diagonals, creating a 3D figure. What's the volume of this figure, in cubic units?
A. $\frac{16\pi\sqrt{2}}{3}$ B. $\frac{32\pi\sqrt{2}}{3}$ C. $16\pi\sqrt{2}$ D. $\frac{64\pi\sqrt{2}}{3}$ E. NOTA

5. Let k be the positive answer to this question. What is $\sqrt{4k + 5}$?
A. 1 B. 5 C. 11 D. Not enough info E. NOTA

6. What does $y = \pm\sqrt{x^2 + \sqrt{x^2 + \sqrt{x^2 + ...}}}$ form in the Cartesian plane?
A. Ellipse B. Hyperbola C. Two ellipses D. Two hyperbolas E. NOTA

7. Find the area of the ellipse defined by $x^2 - 4x + 9y^2 - 36y = 5$. (Hint: $A = \pi ab$)
A. $\frac{45\pi}{4}$ B. 45π C. $\frac{225\pi}{4}$ D. 225π E. NOTA

8. Define the *easiness* of a test to be the product of its median score and winning score divided by its *placing spread* , which is the positive difference between the lowest placing score and the winning score. If a test is won with a 115 and the lowest placing score is 75, find the sum of the digits of the median if the median is 105 less than the easiness.
A. 9 B. 10 C. 11 D. 13 E. NOTA

9. $f(x) = \dfrac{2x^3 + 1}{x^2 + 3x + 1}$ becomes asymptotic to which line as x grows large?
A. $y = 2x - 6$ B. $y = 2x - 3$ C. $y = 2x$ D. $y = 2x + 6$ E. NOTA

10. Find the sum of all x that satisfy $2\sin^2(x) - \sin(x) = 1$, given that $0 < x \leq 2\pi$.
A. $\frac{\pi}{2}$ B. $\frac{5\pi}{3}$ C. 3π D. $\frac{7\pi}{2}$ E. NOTA

11. Let $f(x) = \dfrac{x^3 - 1}{x^3 - x^2}$. If $f(x)$ has V vertical asymptotes and H horizontal asymptotes, what is the value of $V^2 - H^2$?

A. 0 B. 1 C. 3 D. 8 E. NOTA

12. Given that $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$, evaluate $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(2k-1)^2}$.

A. $\frac{\pi^2}{24}$ B. $\frac{\pi^2}{12}$ C. $\frac{\pi^2}{9}$ D. $\frac{\pi^2}{8}$ E. NOTA

13. Let A and R be two sets. $A \cup R$ has 19 elements and $A \cap R$ has 9 elements. Which of the following could be the number of elements in A and R, respectively?

A. 8 and 20 B. 9 and 10 C. 12 and 16 D. 18 and 19 E. NOTA

14. Evaluate $\sin(\arctan(2)) + \cos(\arctan(2))$, where arctan has the traditional restricted range.

A. $\frac{\sqrt{5}}{5}$ B. $\frac{2}{5}$ C. $\frac{3\sqrt{5}}{5}$ D. $\frac{4}{5}$ E. NOTA

15. Eight people, including Eli and Pratik, sit at a circular table to discuss national security. If Eli and Pratik sit next to each other, then war will break out. Find the probability that war doesn't break out, given that any arrangement of these eight people is equally likely.

A. $\frac{2}{7}$ B. $\frac{3}{4}$ C. $\frac{6}{7}$ D. $\frac{7}{8}$ E. NOTA

16. Let $f(x) = \sin(x)$, $g(x) = x^4$, and $h(x) = x\cos(x)$. How many of the following functions are even?

I. $f(g(x))$ II. $f(h(x))$ III. $g(h(x))$ IV. $f(g(h(x)))$ V. $h(f(x))$ VI. $h(x)$

A. 1 B. 2 C. 4 D. 5 E. NOTA

17. If $k = e\sqrt{e\sqrt[3]{e\sqrt[4]{e\sqrt[5]{e\sqrt[6]{\ldots}}}}}$, what is $\ln(k)$? (Hint: $\displaystyle\sum_{k=0}^{\infty} \frac{1}{k!} = e$)

A. $e - 2$ B. $e - 1$ C. e D. $e + 1$ E. NOTA

18. A square matrix is singular (noninvertible) if one row can be written as a linear combination of the other rows. Consider the matrix $Q = \begin{bmatrix} 2 & 7 & 8 \\ 3 & 1 & 4 \\ 1 & 13 & 12 \end{bmatrix}$. Let R_1 denote the first row, R_2 the second row, and R_3 the third row. If $aR_1 + bR_2 = R_3$, what is $a|a| + b|b|$? (that is, to find a and b, you need to solve the equations $2a + 3b = 1, 7a + b = 13$, and $8a + 4b = 12$)

A. 3 B. 5 C. 7 D. 9 E. NOTA

19. Find the sum of the real values of x which satisfy $\log_2(x) + 2\log_4(x-4) = 2\log_8(x^2 - 4x) + \log_8(5)$.

A. 5 B. 6 C. 8 D. 9 E. NOTA

20. I like playing tricks on insects. I have a rubber band that's 3 inches in length and an ant that's currently at one end of the rubber band. The ant walks at a constant speed of one inch per minute. Each time the ant has walked one inch, however, I stretch the rubber band (uniformly) by one inch. It takes the ant k minutes to get to the end. On what interval does k lie?

A. $3 < k < 4$ B. $4 < k < 5$ C. $5 < k < 6$ D. $6 < k < 7$ E. NOTA

21. Consider the following "proof" that $1 = -1$:

Step I: -1 = -1
Step II: $\frac{-1}{1} = \frac{1}{-1}$
Step III: $\sqrt{\frac{-1}{1}} = \sqrt{\frac{1}{-1}}$
Step IV: $\frac{\sqrt{-1}}{\sqrt{1}} = \frac{\sqrt{1}}{\sqrt{-1}}$
Step V: $\frac{i}{1} = \frac{1}{i}$
Step VI: $\frac{i \times i}{1} = \frac{1 \times i}{i}$
Step VII: $-1 = 1$

The first flaw in the proof appears where?

A. Step III B. Step IV C. Step V D. Step VI E. NOTA

22. Find the range of $f(x) = x - [x]$. Here, $[x]$ denotes the greatest integer $\leq x$.

A. $(0, 1)$ B. $[0, 1]$ C. $(-1, 1)$ D. $[0, 1)$ E. NOTA

23. Find the range of $f(x) = \sin^2(x) - \sin^4(x)$.

A. $[0, \frac{1}{4}]$ B. $[0, \frac{1}{4})$ C. $[0, \frac{1}{2}]$ D. $[0, \frac{1}{2})$ E. NOTA

24. Consider the parametric equations $x = \sin(t), y = \cos(2t)$. When graphed, what conic is this a part of?

A. Ellipse B. Hyperbola C. Circle D. Parabola E. NOTA

25. Evaluate $\sin(72°)$. Use the sine values on the last page as necessary.

A. $\frac{\sqrt{5}-1}{4}$ B. $\frac{\sqrt{2(5+\sqrt{5})}}{4}$ C. $\frac{\sqrt{5(5-2\sqrt{5})}}{5}$ D. $2 - \sqrt{3}$ E. NOTA

26. In calculus, we often make trigonometric substitutions to help solve equations. Use the substitution $x = \sin(\theta)$ to find a value of x that satisfies $6x - 8x^3 = 1$ ($\sin(3x) = 3\sin(x) - 4\sin^3(x)$).

A. $\sin(\frac{\pi}{18})$ B. $\sin(\frac{\pi}{10})$ C. $\sin(\frac{\pi}{9})$ D. $\sin(\frac{2\pi}{9})$ E. NOTA

27. The line $y = x + c$ intersects the circle $x^2 + y^2 = 1$ at exactly one point. c has two possible values, A and R. What is AR?

A. -4 B. -2 C. 2 D. 4 E. NOTA

28. Let $f(x) = \dfrac{x+3}{x-1}$. Find $\dfrac{f(2)}{f^{-1}(2)}$ (where $f^{-1}(x)$ denotes the inverse of $f(x)$).

A. $\frac{1}{5}$ B. 1 C. 10 D. 25 E. NOTA

29. Let n be a positive integer. Suppose we plot the points $e^{\frac{2k\pi i}{n}}$ for $k = 0, 1, 2, ..., n-1$ in the complex plane. Which of the following statements are true?
I. These points define a regular n-gon
II. These points are the nth roots of 1
III. These are all distinct points on a unit circle
A. I, II only B. II, III only C. I, III only D. II only E. NOTA

30. Find the sum of the distinct zeros of $f(x) = x^3 + x^2 - 16x + 20$.
A. -20 B. -3 C. -1 D. 16 E. NOTA

Common trigonometric values:
$\sin(0) = 0$ $\sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$ $\sin\frac{\pi}{10} = \frac{\sqrt{5}-1}{4}$ $\sin\frac{\pi}{6} = \frac{1}{2}$ $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$
$\sin\frac{5\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$ $\sin\frac{\pi}{2} = 1$