Round 1 **THETA REGION BOWL** FAMAT State Convention 2009

1. If $u_1 = i+1$ and $u_{n+1} = i \cdot u_n + 1$ where $i = \sqrt{-1}$, find u_{27}.

2. Find the value of $a+b$ if the complex number $\frac{3-i}{5+2i}$ is expressed in the form $a+bi$ where a and b are real numbers.

3. Find $x+y$ if $\left(\frac{1+i}{1-i}\right)^2 + \frac{1}{x+2yi} = 1+i$ and x and y are real numbers.

4. If z is a solution to the equation $x(x+2) = -2$ over the complex numbers, find $|z|$.

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1. The solution set in interval notation for $\frac{2}{x-1} > \frac{5}{3x-7}$ is $(a,b)\cup(c,\infty)$. Find abc.

2. Find the solution set in interval notation for $x < x^2 + 8 < 5x + 2$.

3. How many positive integral factors of 2,520 are even?

4. Find all values of x such that $|x+3| > 2x-3$. Express the solution set in interval notation.

Round 2 **THETA REGION BOWL** FAMAT State Convention 2009

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1. Locate the point that is $\frac{2}{3}$ of the distance from $(2,4)$ to $(8,10)$ and state its distance from the origin.
2. Find the distance between the lines $4x-3y=5$ and $8x-6y=-1$.
3. Find the slope of a line that intersects the curve $x+2xy-3y=4$ at $(1,a)$ and $(2,b)$.
4. A point moves so that its distance from the line $x+y+1=0$ and its distance from the point $(-2,-1)$ are always equal. If the equation of this locus is written in the form $x^2+Bxy+Cy^2+Dx+Ey+F=0$, what is the value of D?

Round 3 **THETA REGION BOWL** FAMAT State Convention 2009

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Round 4 **THETA REGION BOWL** FAMAT State Convention 2009

1. Find the equation of the directrix of the conic section defined by $y^2+6y+8x+25=0$.

2. Find the center of the circle whose diameter is the latus rectum of the parabola $4y=x^2-3x+1$.

3. An ellipse has foci located at $(0,1)$ and $(4,1)$ and it has a major axis of length 6. If the equation of the ellipse is written in the form $A(x-h)^2+B(y-k)^2=AB$, find AB if A and B are relatively prime integers.
4. The vertices of a hyperbola are $(2,\pm 3)$ and the foci are $(2,\pm 5)$. Find the product of the slopes of its asymptotes.

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Round 5 **THETA REGION BOWL** FAMAT State Convention 2009

1. If $f(2x+1) = 4x^2 + 2$ for all x, find $a+b+c$ where $f(x) = ax^2 + bx + c$.

2. If $f(x) = \dfrac{x+1}{x}$, $f(g(x)) = x$, and $g(x) = \dfrac{a}{ax+1}$, find a if $x \notin \{0,1\}$.

3. If $f(x) = x^2 - 7x + 6$, $g(x) = x + 1$, and $f(x) = h(g(x))$, find $h(x)$.

4. If $f(x) = ax^3 + bx^2 - 4x + d$, $f(1) = -12$, $f(-1) = -6$ and $f(0) = -12$, find ab.

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Round 6 **THETA REGION BOWL** FAMAT State Convention 2009

1. If $f(x)=\ln(x^2)$ for $x<0$, find $f^{-1}(x)$.

2. Find x such that $b^{\log_b(x+1)-2\log_b x}=1$ and $b>0, b\neq 1$.

3. For what real value(s) of x does $e^{2x}+e^{x+1}=6e^2$? Express your answer in the form $a+\ln b$ where a and b are real numbers.

4. Find the set of real values of x such that $x^{\frac{1}{\ln x}}=e$. Express the result in interval notation.

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1. For what value(s) of k does the matrix $\begin{bmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{bmatrix}$ have no inverse matrix?

2. If $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} -1 & 1 & 3 \\ 0 & 1 & 2 \end{bmatrix}$, find the entry in the 3rd row, 2nd column of the matrix $(AC - BC)^T$.

3. If matrix X has only one column containing the natural numbers from 1 to 30 inclusive, find the sum of the entries in the matrix $X \cdot X^T$.

4. Find the slope of the line defined by $\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ -1 & 4 & 1 \end{vmatrix} = 0$, where $|M|$ denotes the determinant of matrix M.

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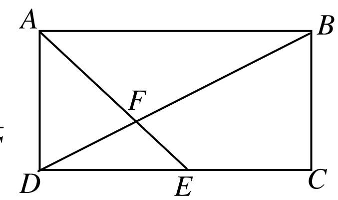
Round 8 **THETA REGION BOWL** FAMAT State Convention 2009

1. The radii of the two concentric circles shown are 3 and 5. If a 40^{o} sector of the larger circle is removed, what is the area of the remaining shaded region?

2. In rectangle *ABCD*, $\overline{AE}$ bisects $\angle BAD$. Find *BF* if *DE* = 6 and *EC* = 2.



Use for part 2 only.

3. In acute ΔABC, *BC* = 20 and the altitude to $\overline{BC}$ is 15. Find the area of the trapezoid formed by a line segment joining $\overline{AB}$ and $\overline{AC}$ that is parallel to $\overline{BC}$ and 6 units from *A*.

4. A square piece of paper is folded along its diagonal. If the resulting figure has a perimeter of 6 inches, find the area of the original square piece of paper in square inches.

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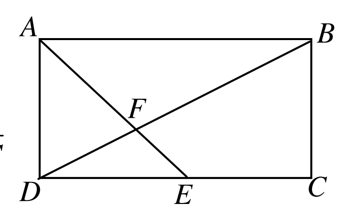

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1. The sum of the first 3 terms of an arithmetic sequence with positive terms is 18. The geometric mean between the first and third terms is 4. Find the sum of the 4th and 5th terms of the sequence.

2. Find the sum of the infinite series $1+6-\frac{1}{2}+2+\frac{1}{4}+\frac{2}{3}-\frac{1}{8}+\frac{2}{9}.....$

3. One root of $mx^2-8x+m+1$ is three times the other root. Find the value(s) of m.

4. A quadratic expression ax^2+bx+c has the value 3 when $x=0$ and the value 1 when $x=2+\sqrt{2}$ or $x=2-\sqrt{2}$. Find abc.

Round 9 **THETA REGION BOWL** FAMAT State Convention 2009

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Round 10 **THETA REGION BOWL** FAMAT State Convention 2009

1. Find the middle term of the expansion of $\left(\frac{3}{\sqrt[3]{x}}-\frac{\sqrt{x}}{3}\right)^{10}$ in simplified radical form.

2. In how many ways can 5 people be seated around a circular table if two particular persons must be seated side by side?

3. One bag contains 3 red, 2 orange, and 2 white marbles. Another bag contains 1 red, 1 orange, and 3 white marbles. If one marble is drawn at random from each bag, find the probability that either both marbles are orange or neither marble is orange.

4. The equation $x^3-4x^2+mx+p^2=0$ has the sum of two of its roots equal to zero. Find $\frac{p^2}{m}$.

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