FAMAT February Regional Statistics Team Test Solutions

1. $1 = a + b + c$
   $E(X) = 2 = a + 2b + 3c$
   $Var(X) = 1/2 = (1-2)^2 a + (2-2)^2 b + (3-2)^2 c$
   $\begin{cases} a+b+c=1 \\ a+2b+3c=2 \\ a+c=1/2 \end{cases} \Rightarrow a=\frac{1}{4}, b=\frac{1}{2}, c=\frac{1}{4} \Rightarrow a-b+c = \boxed{0}$

2. $A = \sigma_{x+y} = \sqrt{\sigma_x^2 + \sigma_y^2} = \sqrt{16^2 + 20^2} \approx 25.61$
   $B = \sigma_{4x-8y} = \sqrt{4^2 \cdot \sigma_x^2 + 8^2 \cdot \sigma_y^2} = \sqrt{16 \cdot 16^2 + 64 \cdot 20^2} \approx 172.33$
   $C = \mu_{2x-y} = \mu_{2x} - \mu_y = 2 \cdot 94 - 84 = 104$
   $D = \sigma^2_{6x+2y} = 6^2 \cdot \sigma_x^2 + 2^2 \cdot \sigma_y^2 = 36 \cdot 16^2 + 4 \cdot 20^2 = 10816$
   $A + B + C + D = 25.61 + 172.33 + 104 + 10816 = \boxed{11117.94}$

3. $2a + b = 12,\ 1^2 \cdot a^2 + (1/2)^2 b^2 = 20 \Rightarrow a = 2, b = 8 \text{ or } a = 4, b = 4 \Rightarrow \boxed{a - b = -6 \text{ or } a - b = 0}$

4. $MOE = z^* \cdot \frac{\sigma}{\sqrt{n}} \Rightarrow 0.5 \geq 1.96 \frac{4}{\sqrt{n}} \Rightarrow n \geq 245.86 \Rightarrow C = 246$
   $A = \sqrt{2}, B = 2 \Rightarrow \boxed{ABC = 492\sqrt{2}}$

5. Let S indicate actually taking steroids and N indicate not actually taking steroids. Let + indicate a positive steroid test and – a negative steroid test.
   $A = P(N \mid +) = \frac{P(N \cap +)}{P(+)} = \frac{P(+ \mid N)P(N)}{P(+ \mid N)P(N) + P(+ \mid S)P(S)} = \frac{0.05(0.80)}{0.05(0.80) + 0.95(0.20)} = \frac{4}{23}$
   $B = P(S \mid -) = \frac{P(S \cap -)}{P(-)} = \frac{P(- \mid S)P(S)}{P(- \mid N)P(N) + P(- \mid S)P(S)} = \frac{0.05(0.20)}{0.95(0.80) + 0.05(0.20)} = \frac{1}{77}$
   $\boxed{AB = \frac{4}{1771}}$

6. Let x equal the length of leg one and y equal the length of leg two.
   $E[x] = E[y] = \frac{1}{n} \cdot 1 + \frac{1}{n} \cdot 2 + \ldots + \frac{1}{n} \cdot n = \frac{1}{n}(1 + 2 + \ldots + n) = \frac{1}{n}\left(\frac{n(n+1)}{2}\right) = \frac{n+1}{2}$
   $E[Area_T] = E\left[\frac{1}{2}xy\right] = \frac{1}{2}E[x]E[y] = \frac{1}{2} \cdot \frac{n+1}{2} \cdot \frac{n+1}{2} = 8 \boxed{\Rightarrow n = 7}$

7. $P(X \leq 2) = 1 - P(X \geq 3) = 1 - P(X = 3) - P(X = 4) - P(X = 5) =$
   $1 - {}_5C_3 p^3 (1-p)^2 - {}_5C_4 p^4 (1-p) - {}_5C_5 p^5 = -6p^5 + 15p^4 - 10p^3 + 1 \Rightarrow \boxed{A + B + C + D + E + F = 0}$

8. $A = P(A \cap B) = -P(A \cup B) + P(A) + P(B) = -2a + a + b = b - a$

$B = P(A|B') = \dfrac{P(A \cap B')}{P(B')} = \dfrac{P(A) - P(A \cap B)}{1-b} = \dfrac{a-(b-a)}{1-a} = \dfrac{2a-b}{1-b}$

$C = P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{b-a}{b}$

$D = \dfrac{1}{2}$ because if a was greater than 0.5 than B would be greater than 1

$$\frac{AB(1-b)-4Da(b-a)}{C} = \frac{(b-a)\left(\dfrac{2a-b}{1-b}\right)(1-b) - 4\dfrac{1}{2}a(b-a)}{\left(\dfrac{b-a}{b}\right)} = \boxed{-b^2}$$

9. $\boxed{\text{CACABB} \Rightarrow 90}$

10. In order for the chi-squared goodness of fit test to not be significant at the 5% level, the chi-squared statistic must be less than 11.07.

$$\chi^2 = \frac{1}{20}\left((18-20)^2 + (19-20)^2 + (21-20)^2 + (22-20)^2 + (a-20)^2 + (40-a-20)^2\right) < 11.07$$

$$\Rightarrow \frac{1}{20}\left(4+1+1+4+2(a-20)^2\right) < 11.07 \Rightarrow \frac{1}{10}a^2 - 4a + 29.43 < 0 \Rightarrow 9.7 < a < 30.2$$

$\boxed{\Rightarrow x = 10,\ y = 30}$

11. Statements I, II, and III are true. Statement IV is false as blocking divides subjects into groups to reduce variation and does not substitute as a control group. $\boxed{3}$

12. $A = P(W|3) = \dfrac{1}{4}$

$$B = P(1|R) = \frac{P(1 \cap R)}{P(R)} = \frac{P(R|1)P(1)}{P(R|1)P(1) + P(R|2)P(2) + P(R|3)P(3)}$$

$$= \frac{(1/4)(1/3)}{(1/4)(1/3) + (1/2)(1/3) + (3/4)(1/3)} = \frac{1}{6}$$

$$C = P(2|R') = \frac{P(2 \cap W)}{P(W)} = \frac{P(W|2)P(2)}{P(W|1)P(1) + P(W|2)P(2) + P(W|3)P(3)}$$

$$= \frac{(1/2)(1/3)}{(3/4)(1/3) + (1/2)(1/3) + (1/4)(1/3)} = \frac{1}{3}$$

$$D = P(W) = P(W|1)P(1) + P(W|2)P(2) + P(W|3)P(3) = (3/4)\frac{1}{3} + (1/2)\frac{1}{3} + (1/4)\frac{1}{3} = \frac{1}{2}$$

$$A + B + C + D = \frac{1}{4} + \frac{1}{6} + \frac{1}{3} + \frac{1}{2} = \boxed{\frac{5}{4}}$$

13. The number of die that turn up even is a binomial variable, so it is also approximately normal with $\mu = 100 \cdot \frac{1}{2} = 50$ and $\sigma = \sqrt{100 \cdot \frac{1}{2}\left(1 - \frac{1}{2}\right)} = 5$.

$$P\left(\frac{39.5 - 50}{5} < z < \frac{65.5 - 50}{5}\right) = P(-2.1 < z < 3.1) = P(z < 3.1) - P(z < -2.1) \approx \boxed{0.981}$$

14. Let the hypotenuse equal h and the radius equal r.

$$A = E[Area_T] = E\left[\frac{1}{2}\left(\frac{h}{\sqrt{2}}\right)^2\right] = \frac{1}{4}E[h^2] = \frac{1}{4}\left(Var[h] + (E[h])^2\right) = \frac{1}{4}(4 + 100) = 26$$

$$B = E[Area_C] = E[\pi r^2] = \pi E[r^2] = \pi\left(Var[r] + (E[r])^2\right) = \pi(9 + 81) = 90\pi$$

$$\boxed{AB = 2340\pi}$$

15. A is false trials are independent, B is true, C is false there are two possible outcomes, D is false probability of success stays the same on all trials

A + B + C + D = 2

$\boxed{2! = 2}$