Team Round Solutions

1. $-\frac{\sqrt{3}}{2}\bullet\frac{\sqrt{3}}{3}\bullet\sqrt{2}\bullet\frac{2\sqrt{3}}{3}\bullet 1\bullet -1=\frac{\sqrt{6}}{3}$

2.
$$\frac{\sin\theta\left(\frac{\sin\theta}{\cos\theta}\right)+\cos\theta}{\cos\theta}\left(\frac{\frac{1}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}}\right)\left(\frac{\tan\theta\csc^2\theta}{\sec^2\theta}\right)$$
$$\left(\tan^2\theta+1\right)\left(\frac{\sin\theta}{\sin^2\theta+\cos^2\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right)$$
$$\sec^2\theta(\sin\theta)(\cot\theta)$$
$$\sec\theta$$

3. (-9) False, the inverse is $f^{-1}(x)=\frac{1}{5}\ln x$

(2) True, change of base

(20) True

$(4e^x+3)(4e^x-2)=0$

(15) True $e^x=-\frac{3}{4},\ e^x=\frac{1}{2}$

$x=\ln\frac{1}{2}=-\ln 2$

(-3) False

(32) False, the function is even. Therefore, 2+20+15=**37**

4. The angle $\frac{31\pi}{12}$ is coterminal to $\frac{7\pi}{12}$. Also since $\frac{7\pi}{12}$ to $\frac{7\pi}{6}$, we can use the half angle identify for cosine.

$$\cos\left(\frac{31\pi}{12}\right)=\cos\left(\frac{7\pi}{12}\right)=-\sqrt{\frac{1+\cos\frac{7\pi}{6}}{2}}=-\sqrt{\frac{1+-\frac{\sqrt{3}}{2}}{2}}=-\frac{1}{2}\sqrt{2-\sqrt{3}}$$
$$\therefore ABC=-\frac{1}{2}(2)\left(-\sqrt{3}\right)=\sqrt{3}$$

5. A: The possible real zeros are $\frac{\pm1\pm3}{\pm1\pm2\pm3\pm6}$. The smallest is -3.

B: 4 zeros

C: -2 is the smallest zero, the multiplicity is 3

D: -3; because the bottom row of synthetic division alternates signs

E: 4 turns

F: $\frac{-b}{2a}=\frac{--1}{2(-2)}=-\frac{1}{4}$ $\quad\therefore A+B+C+D+E+F=-3+4+3+-3+4+-\frac{1}{4}=\frac{19}{4}$ or 4.75

$$\frac{\log_3 x}{\log_3 27}+\frac{\log_3 x}{\log_3 3}+\frac{\log_3 x}{\log_3 9}=11$$
$$\frac{\log_3 x}{3}+\frac{\log_3 x}{1}+\frac{\log_3 x}{2}=11$$

6. $2\log_3 x+6\log_3 x+3\log_3 x=66$

$\log_3 x=6$

$x=729$

7. A: 8

B: $e^{-270^\circ}=e^{\frac{-3\pi}{2}}\quad\therefore\frac{\pi}{2}$

C: $\left(2cis\frac{11\pi}{6}\right)^6=64cis11\pi\ \ \therefore 64$

D: unity is the complex number $1+0i$, therefore the angle is 0 rad or 0°.

A+B+C+D= $72+\frac{\pi}{2}$

8. A: $(x+10)^2=4y$, the length of the latus rectum is $4p=4(1)=4$

B: $e=\frac{c}{a}=\frac{5}{3}$

C: $y-2=\pm\frac{\frac{4}{1}}{\frac{2}{3}}(x-2),\ \ \therefore \textit{the positive slope is } 6$

D: $\frac{(x-4)^2}{36}+36y^2=1$, the length of the major axis of the ellipse is $2a=2(6)=12$

$\therefore\frac{18}{12}+\frac{5}{3}(6)+4=\frac{31}{2}\ \textit{or}\ 15.5$

9. A: $\left\langle\frac{3}{5},\frac{4}{5}\right\rangle$

B: $\langle 2,10\rangle$

C: $\langle 10\cos 60^\circ,10\sin 60^\circ\rangle=\langle 10,10\sqrt{3}\rangle$

D: -12+12=0

$$DA+B\times C=0\left\langle\frac{3}{5},\frac{4}{5}\right\rangle+\langle 2,10\rangle\times\langle 10,10\sqrt{3}\rangle=\begin{vmatrix} i & j & k\\ 2 & 10 & 0\\ 10 & 10\sqrt{3} & 0\end{vmatrix}=\left(20\sqrt{3}-100\right)k\ \ or\ \ \left\langle 0,0,20\sqrt{3}-100\right\rangle$$

$(y+3)^2-(y+3)(y-1)-2(y-1)^2=0$

10. $\left((y+3)+(y-1)\right)\left((y+3)-2(y-1)\right)=0$

$y=-1,\ 5$

11. A: $\frac{3z-5-2(z+2)}{z+2} \le 0 \rightarrow \frac{z-9}{z+1} \le 0$, Therefore the first positive integer is **1**.

$(-1,9]$

B: $f(x)=\frac{x(x-2)(x+2)}{x(x+2)(x^2-2x+4)}=\frac{x-2}{x^2-2x+4}$, The only zero is **2**.

C: **-4**

D: $f(x)=\frac{(2x-1)(x-2)}{(x+2)(x-2)}$, the ordinate for the removable discontinuity is **2**.

AB+BC+CD = **-14**

12.

$$\sin\theta\cos\theta+\frac{1}{2}=0$$

$$\frac{1}{2}\sin 2\theta=-\frac{1}{2}$$

$$\sin 2\theta=-1$$

$$2\theta=\frac{3\pi}{2},\frac{7\pi}{2} \qquad 2\theta\in[0,\pi)$$

$$\theta=\frac{3\pi}{4},\frac{7\pi}{4}$$

$$\therefore \frac{3\pi}{4}+\frac{7\pi}{4}=\frac{10\pi}{4}=\frac{5\pi}{2}$$

13. There are 17 prime numbers between 1 and 60, the only containing a 9 are 19, 29, & 59. , So, the probability of choosing one of these prime numbers is $\frac{3}{17}$.

14. $\det \mathbf{C}=8-6=2$

$$\mathbf{AB}=\begin{bmatrix}4 & 6\\ 2 & -15\end{bmatrix}$$

$$\mathbf{C}^{-1}=\begin{bmatrix}1 & -\frac{1}{2}\\ -3 & 2\end{bmatrix}$$

$$2\begin{bmatrix}4 & 6\\ 2 & -15\end{bmatrix}+\begin{bmatrix}1 & -\frac{1}{2}\\ -3 & 2\end{bmatrix}+\begin{bmatrix}2 & -3\\ 0 & 6\end{bmatrix}=\begin{bmatrix}11 & \frac{17}{2}\\ 1 & -22\end{bmatrix}$$

15. A: 4 petals

B: 3

C: 3

D: $\frac{3}{2}$

$$\sqrt{4(3)(3)(3/2)}=\sqrt{54}=3\sqrt{6}$$