

1. Total revenue from gum = $260g^2 - 6g^3 + 1 \rightarrow 520g - 18g^2 = 0 \rightarrow g = \frac{260}{9} \rightarrow Y = 29$

Total revenue from bread = $390b^2 - 5b^3 + 15 \rightarrow 780b - 15b^2 = 0 \rightarrow b = 52 \rightarrow A = 52$

Total revenue from candy = $250c^2 - 20c^3 \rightarrow 500c - 60c^2 = 0 \rightarrow c = \frac{25}{3} \rightarrow L = 8$

Total revenue from soda = $15s^3 - 190s^2$. Note that as s increases, however, this function increases; it has no global maximum, and so $E = 1337$.

$Y + A + L + E = 1426$.

2. $f'(x) = 6x + 2$; $f'(1) = 8 \rightarrow m$ is the line $y = 8x + 1$. $g'(x) = 3x^2 - 6x$; $g'(2) = 0 \rightarrow$ the line normal to g will be the vertical line $x = 2$. $x = 2$ intersects $y = 8x + 1$ at the point $(2, 17)$.

3. The first region will have volume $\frac{4}{3}\pi(R^3 - r^3)$; hence, $\frac{dV}{dt} = \frac{4}{3}\pi(3R^2\frac{dR}{dt} - 3r^2\frac{dr}{dt})$. When $R = r$, we have $R = 7$; hence, we get $4\pi(7^2)(\frac{dR}{dt} - \frac{dr}{dt}) = 196\pi(-2 - 1) = -588\pi = A$.

The second region will have volume $S^3 - s^3$; hence, $\frac{dV}{dt} = 3S^2\frac{dS}{dt} - 3s^2\frac{ds}{dt}$. When $S = s$, we have $S = 6$; thus, we get $3(6^2)(-2 - 1) = -324 = B$

Thus, $\frac{A}{B} = \frac{49\pi}{27}$.

4. $\frac{16!}{13!3!} = \frac{16(15)(14)}{6} = 8(5)(2)(7) = 2^4(5)(7)$; $A = 7$

Part B can be done with solids of revolution or with Pappus' Theorem, but note that the solid of revolution is a cylinder of radius $8\sqrt{3}$ and height 1, but with a cylindrical chunk of radius 1 cut out; thus, $V = \pi(8\sqrt{3})^2 - \pi = 191\pi \rightarrow B = 191$

$7 \times 191 = 1337$.

5. We first need to compute the length of Grant's path; it's a quarter-circle of radius 4, so $\frac{1}{4}(2\pi)(4) = 2\pi$.

Thus, the total amount of time t_0 he will take can be solved for by setting $\frac{\pi^2}{4} \int_0^{t_0} |\sin\left(\frac{\pi t}{4}\right)| dt = 2\pi$;

$t_0 = 4$. Hence, we need Ari to take exactly 4 seconds. Suppose Ari is torpedoed while at the point $(-4 + k, k)$. Then his total x -distance will stay 4, but his total y -distance will become $4 + 2k$. Since his speed is a constant $2\sqrt{2}$ units/sec, the time it will take him is $t = \frac{d}{v} = \frac{\sqrt{4^2 + (4+2k)^2}}{2\sqrt{2}}$. We know that $t = 4$,

so cross-multiplying and squaring both sides, we obtain $(8\sqrt{2})^2 = 128 = 16 + 4k^2 + 16k + 16$

$\rightarrow 4(k^2 + 4k - 24) = 0 \rightarrow k = \frac{-4 \pm \sqrt{112}}{2} = -2 + 2\sqrt{7}$.

6. By definition, if f is not concave up then $f''(x) \leq 0$; hence, $A = 0$.

$$B = \int_0^{\frac{\pi}{3}} \frac{\sin^3(x)}{\cos^2(x)} dx = \int_0^{\frac{\pi}{3}} \frac{\sin(x)(1 - \cos^2(x))}{\cos^2(x)} dx = \int_0^{\frac{\pi}{3}} (\sec(x)\tan(x) - \sin(x)) dx =$$

$$\sec\left(\frac{\pi}{3}\right) - \sec(0) + \cos\left(\frac{\pi}{3}\right) - \cos(0) = \frac{1}{2}$$

$$A + B = \frac{1}{2}.$$

7. Using L'Hopital's Rule on A and C , and factoring the numerator of B into $(x - 1)^3$ yields:

$$A = \lim_{x \rightarrow 3} \frac{\frac{1}{2\sqrt{x}}}{\frac{1}{\sqrt{2x+3}}} = \frac{\sqrt{3}}{2}$$

$$B = \lim_{x \rightarrow 1} (x - 1)^2 = 0$$

$$C = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$$

$$A + B + C = \frac{2\sqrt{3}}{3}$$

$$8. A - R = \int_{-1}^1 \left(\frac{x^3}{x^2 - x + 1} - \frac{1}{x - x^2 - 1} \right) dx = \int_{-1}^1 \left(\frac{x^3}{x^2 - x + 1} + \frac{1}{x^2 - x + 1} \right) dx =$$

$$\int_{-1}^1 \frac{x^3 + 1}{x^2 - x + 1} dx = \int_{-1}^1 \frac{(x + 1)(x^2 - x + 1)}{x^2 - x + 1} dx = \int_{-1}^1 (x + 1) dx = 2.$$

9. We know that $f(\frac{\pi}{4}) = 0$ and $g(\frac{\pi}{4}) = 0$. Using this, and the Fundamental Theorem of Calculus, we can approximate $f(x+h) = f(x) + f'(x)h$. $f'(x) = \frac{\sin(x)}{x}$; $g'(x) = \frac{\cos(x)}{x}$. Thus:

$$A + B = \frac{\sin(\frac{\pi}{4})}{\frac{\pi}{4}} \left(\frac{\pi}{4} + \frac{\pi}{12} \right); C + D = \frac{\cos(\frac{\pi}{4})}{\frac{\pi}{4}} \left(\frac{\pi}{4} + \frac{\pi}{12} \right)$$

$$A + B + C + D = \frac{4\sqrt{2}}{3}.$$

10. $2B + 3M = 500 \rightarrow B = 250 - \frac{3M}{2}$. Thus Aneesh's utility is $M^2(250 - 1.5M)^3$; maximizing, $2M(250 - 1.5M)^3 + M^2(3)(250 - 1.5M)^2(-1.5) = 0 \rightarrow M(250 - 1.5M)^2(2(250 - 1.5M) - 4.5M) = 0 \rightarrow 500 - 7.5M = 0 \rightarrow M = \frac{200}{3} \rightarrow 2B = 300 \rightarrow B = 150$.

11. Suppose Girish travels x yards on one side of the street. Then his total diagonal distance will be $\sqrt{(100-x)^2 + 30^2} = \sqrt{(100-x)^2 + 900}$. Since time is distance divided by velocity, his total time taken will be $\frac{x}{5} + \frac{\sqrt{(100-x)^2 + 900}}{3}$. We want to minimize this; thus, $\frac{dt}{dx} = \frac{1}{5} + \frac{1}{3} \left(\frac{x-100}{\sqrt{(100-x)^2 + 900}} \right)$. Setting this equal to 0, we get $-\frac{3}{5} = \frac{x-100}{\sqrt{(100-x)^2 + 900}}$. Cross-multiplying and squaring, we get $25(100-x)^2 = 9(100-x)^2 + 8100 \rightarrow 16(100-x)^2 = 8100 \rightarrow 4(100-x) = 90 \rightarrow x = \frac{155}{2}$.

12. Letting s be side length and A be area, we have $s = \sqrt[3]{V}$ and $A = 6V^{\frac{2}{3}}$. At $t = 12$, $V = 27$ and

$$\frac{dV}{dt} = \frac{15}{4}. \text{ Thus:}$$

$$\frac{ds}{dt} = \frac{1}{3V^{\frac{2}{3}}} \frac{dV}{dt} = \frac{1}{3(9)} \times \frac{15}{4} = \frac{5}{36}$$

$$\frac{dA}{dt} = 4V^{-\frac{1}{3}} \frac{dV}{dt} = \frac{4}{3} \times \frac{15}{4} = 5$$

$$AR = \frac{25}{36}$$

13. The first square has side length $2r$. ω_2 has radius of $\frac{2r}{2}\sqrt{2} = r\sqrt{2}$, and so the second square has side length $2r\sqrt{2}$; thus, the outer circle has radius $2r$. Since $\frac{dC}{dt} = 2$, we have $2\pi \frac{dr}{dt} = 2 \rightarrow \frac{dr}{dt} = \frac{1}{\pi}$.

$$A = \pi(2r)^2 = 4\pi r^2 \rightarrow \frac{dA}{dt} = 8\pi r \frac{dr}{dt} = 8\pi \left(\frac{1}{\pi} \right) = 8.$$

14. $f'(x) = \cos(\cos(e^x))(-\sin(e^x))(e^x)$; $f'(\ln \frac{\pi}{2}) = \cos(0)(-1)(\frac{\pi}{2})$; $f(\ln(\frac{\pi}{2})) = \sin(0) = 0$; answer $-\frac{\pi}{2}$.

15. We can ignore the x^2 , x , and constant terms; thus, $f'''(x) = 60x^2 - 78x + 18 = 6(10x^2 - 13x + 3) = 6(10x - 3)(x - 1)$; $|1 - \frac{3}{10}| = \frac{7}{10}$.