Calculus Individual—January—SOLUTIONS

1. B—Finding the y-coordinate for $x=\frac{\pi}{4}$ gives $\frac{1}{2}$. $\frac{dy}{dx}=2\sin x\cos x$ and at $x=\frac{\pi}{4}, \frac{dy}{dx}=1$. The only equation with slope 1 and the given y-value is B.

2. A—Using the 2nd Fund. Thm. Of Calculus, we get $5\cos 5x-2\cos 2x$.

3. D— $\int_{-2}^{4} f(x)dx=\int_{-2}^{3} f(x)dx + \int_{3}^{4} f(x)dx \Rightarrow \int_{-2}^{4} f(x)dx-\int_{3}^{4} f(x)dx=\int_{-2}^{3} f(x)dx$.

   So, a – b= $\int_{-2}^{3} f(x)dx$ and $\int_{3}^{-2} f(x)dx=b-a$.

4. D—Think of this as the definition of the derivative $\tan^{-1} x$ at $x=1$. The derivative of $\tan^{-1} x$ is $f^{/}(x)=\frac{1}{1+x^2}$. At $x=1$, you get $\frac{1}{2}$.

5. D—Plugging in -8 yields $\frac{0}{0}$, which is indeterminant. One can either factor and cancel to get $\frac{x-3}{x+2}$, then plug in again to get the answer or use L'Hopital's Rule.

6. E— $\frac{-5\sqrt{3}}{3}$ Using the Mean Value Thm., we get $\frac{f(5)-f(0)}{5-0}=19=3c^2-6$ and $c=\pm\frac{5\sqrt{3}}{3}$, but the only value that works for the given interval is the negative case.

7. A— $\int_{2}^{e+1}\left(\frac{4}{x-1}\right)dx=4\ln|x-1|\Big|_{2}^{e+1}=4$.

8. A— $h^{/}(x)=f^{/}(g(x))g^{/}(x)$. Subbing in *a* for *x* and *g(a)=c*, we get *6b*.

9. B— $\frac{dy}{dx}=1-\left(x\frac{dy}{dx}+y\right)\sin xy$. Plugging in (0, 1), we get $\frac{dy}{dx}=1$. The slope of the normal line is the negative reciprocal, which is -1.

10. B— $f^{/}(x)=\frac{1}{2}(x^3+5x+121)^{-1/2}(3x^2+5)(x^2+x+11)+(x^3+5x+121)^{1/2}(2x+1)$. Plugging in yields the answer.

11. E—225 Integrating, we get $v(t)=-10t+c$. Using the initial condition gives C = 50. Integrating velocity will give $s(t)=-5t^2+50t+c$. Again, plugging in the initial condition gives C = 100. The max height occurs when the velocity = 0 (or *t* = 5). Plugging that in to *s(t)* gives 225.

12. A— $\cos^4 x-\sin^4 x=(\cos^2 x+\sin^2 x)(\cos^2 x-\sin^2 x)=1\left(\cos^2 x-\sin^2 x\right)=\cos 2x$.

    $\arccos(\cos 2x)=\pm 2x+2\pi k$, where k is an integer. Hence, $\frac{dy}{dx}=\pm 2$ and $\frac{d^2y}{dx^2}=0$.

13. C—There is no rule to split the integral of a product.

14. B—Using the quotient rule,

    $$g^{/}(1)=\frac{f(1)[-1-f^{/}(1)]-[-1-f(1)]][f^{/}(1)]}{[f(1)]^2}=\frac{4(-1-2)-(-1-4)(2)}{(4)^2}=\frac{-1}{8}$$

15. D—Using logarithmic differentiation, we get

    $\ln y=e^x\ln\sin x\Rightarrow\frac{1}{y}\frac{dy}{dx}=e^x\ln\sin x+e^x\cot x\Rightarrow\frac{dy}{dx}=(\sin x)^{e^x}(e^x\ln\sin x+e^x\cot x)$. Factor out e^x to get the final answer.

16. B—Differentiating the perimeter, $P=4s$, we get $\frac{dP}{dt}=4\frac{ds}{dt}$. Plugging in $\frac{ds}{dt}=0.4$, we get $\frac{dP}{dt}=1.6$. Using area, $A=s^2=\frac{P^2}{16}\Rightarrow\frac{dA}{dt}=\frac{P}{8}\frac{dP}{dt}\Rightarrow\frac{P}{8}(1.6)=0.2P$.

---

17. B—$\frac{d}{dt}(\pi r^2)=3\left[\frac{d}{dt}(2\pi r)\right]\Rightarrow 2\pi r\frac{dr}{dt}=3\left(2\pi\frac{dr}{dt}\right)\Rightarrow r=3$.

---

18. D—

$$x^2\frac{dy}{dx}+2xy+2x=2y\frac{dy}{dx}\Rightarrow\frac{dy}{dx}=\frac{2xy+2x}{2y-x^2}\Rightarrow @(1,1):\frac{dy}{dx}=4.$$

$$\frac{d^2y}{dx^2}=\frac{(2y-x^2)\left(2x\frac{dy}{dx}+2y+2\right)-(2xy+2x)\left(2\frac{dy}{dx}-2x\right)}{(2y-x^2)^2}$$ and at (1, 1), we get -12.

---

19. D—Avg. Val.= $\frac{1}{e-1}\int_1^e x\ln x\,dx=\left(\frac{x^2}{2}\ln x-\frac{x^2}{4}\right)_1^e=\frac{e^2+1}{4}$ (Int. by Parts). Mult. by $\frac{1}{e-1}$ to get ans.

---

20. A—Using u-substitution, we let $u=\ln(\pi x)$ and $du=\frac{1}{x}dx$. We get $\int u\,du=\frac{u^2}{2}=\frac{\ln^2(\pi x)}{2}\Big|_{1/\pi}^{e^4/\pi}=8$.

---

21. A—$y(p)=ap^2\Rightarrow y'=2ap$. the eqn. of the tangent line is $y=ap^2+2ap(x-p)$ & the x-int. = $\frac{p}{2}$.

---

22. B—Plugging $-\infty$ into the problem gives $\frac{10}{\infty}$ and the limit is 0.

---

23. B—$h'(5)=f(5)g'(5)+g(5)f'(5)=7$.

---

24. A—$h'(4)=\frac{g(4)f'(4)-f(4)g'(4)}{(g(4))^2}=\frac{3(-3)-0}{9}=-1$.

---

25. D—$y'=6x^2-6(k+1)x+6k\Rightarrow y''=12x-6(k+1)=12\left(x-\frac{k+1}{2}\right)$. For $1<x<k$, $x-1>0$; $x-k<0\Rightarrow y'<0$. If $1<x<\frac{k+1}{2}$, $y''<0$. If $\frac{k+1}{2}<x<k$, $y''>0$.

---

26. B—Average velocity $=\frac{\Delta h}{\Delta t}=\frac{h(6)-h(0)}{6-0}=\frac{564-1200}{6}=-106$.

---

27. E—The rate of change of y is $\frac{dy}{dt}=6x^2\frac{dx}{dt}-4\frac{dx}{dt}$. Given $\frac{dx}{dt}=3$ and the point (0, 1), we plug in and get $\frac{dy}{dt}=6(0)^2(3)-4(3)=-12$ cm/sec.

---

28. D—Volume $=\pi\int_0^1(x-x^4)dx=\frac{3\pi}{10}$.

---

29. A—Volume = $\frac{\pi}{4}\int_0^1(\sqrt{x}-x^2)^2dx=\frac{9\pi}{280}$.

---

30. A—Area = $\frac{1}{2}(2)[2+2(3)+2(3)+2(4)+5]=27$.