

2008 FAMAT State Math Convention
Interschool Test

1. It's better to be lucky than good, so let's test your luck with this question. We picked a two-digit number, for each digit you place correctly, you get a half a point.
2. Ramanujan is credited for discovering that 1729 was the smallest number that can be written as the sum of two positive cubes two different ways, ignoring commutability ($12^3 + 1^3 = 10^3 + 9^3$). What is the third smallest number that can be written as the sum of two fourth powers two different ways?
3. Find the next five terms of the sequence 1, 2, 1, 3, 2, 5, 2, 7, 4, 11, 2, 13, 6, 17, 4, 19, 6, 23, 4, 29, 10.
4. Ptolemy, Vitruvius, Fibonacci and Brahmagupta all made pretty good approximations of the same number. Whose approximation was the worst and what was his approximation?
5. Everyday I go to a new restaurant, but I always like to keep track of what I currently consider the best restaurant I've ever been to (before starting this, I didn't have a favorite restaurant). Since I pick the next restaurant to go to randomly, the probability that I like the restaurant I go to on Day X more than the restaurant I go to on Day Y is 0.5. After n days (for some really large n), find a closed form approximation (not a summation) of the number of different favorite restaurants I should expect to have had.
6. Instead of having a calendar, I use two cubes to keep track of the day of the month. Each cube has exactly one digit on each side, but I can express any integer from 01 to 31 (both cubes are used each day). How many of the faces of these two dice have prime numbers on them?
7. What is the one millionth prime?
8. What is the smallest n such that there are at least twenty integers x such that $\phi(x) = n$?
9. Find recurrence relation, in terms of k , for $f_k(x) = \sum_{n=1}^{\infty} \frac{n^k}{x^n}$ where $k \in \{0,1,2,\dots\}$ and $x > 1$.
10. What are the 3rd and 14th digits spoken in the film π (Aronofsky 1998)?
Note: Do not count ordinal numbers, roman numerals, etc.

11. Find $\lim_{n \rightarrow \infty} \frac{\sum_{i=1}^n \sqrt{i}}{n\sqrt{n}}$

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12. I have an unfair coin that will land on heads with probability p . In order to try to figure out how likely p , I decide to flip the coin until I get a heads. I repeat this experiment 1000 times and get the following results:

Tails seen before first head	0	1	2	3	4	5
Number of occurrences	725	199	54	16	4	2

For example, in 725 of the 1000 trials, there were exactly 0 tails before the first heads.

Find, to the nearest thousandth, the value of p that makes the data I found most likely.

13. Fit the following data to a polynomial of lowest degree possible and then find $f(7)$.

x	1	2	3	4	5	6
$f(x)$	7	37	141	391	883	1737

14. Who said “bjzvzj nvijovywm vwxjpvjpmvhwoc hwodywgvzdaadypgod n.vc vdio bmwo nv hkdmdywggt.” and “dvzji'ovx gd q vdivhwoc hwodyn”?
15. What year did the Mu Alpha Theta National Convention t-shirts incorrectly approximate π and, to two significant digits, how much were they off by?
16. Find the equation of the graph representing $y = x^3$ reflected over the line $y = -3x + 1$.
17. 3463 843 786 63 646389 263 6463.
18. Find the ratio of the area of a regular n -gon and the area of the circle circumscribed about that n -gon as a function of n .
19. Find $\lim_{n \rightarrow \infty} \prod_{i=1}^n \sqrt{\frac{i}{n}}$.
20. What is the probability that two random chosen integers are relatively prime?
21. Let t_n be the n^{th} triangular number and s_n be the n^{th} square number such that $t_1 = s_1 = 1$ and $t_n = s_n = 0$ for $n < 1$. Write s_n in terms of elements of t .
22. Find the sum of the odd factors of 6,300,179,600.
23. Evaluate $\sqrt[4]{2^{2^{4^{2^{2^{\dots}}}}}}$

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24. For primes $p > 5$, $\frac{1}{p}$ is a repeating decimal in base 10. What is the longest its shortest period can be?
25. Order the following Florida cities from northernmost to southernmost: The City Beautiful, The Magic City, The Big Guava, City of Palms, The City of Five Flags, and Venice of America
26. What is the speed of light in nails per metric second to three significant figures?
27. A square and an equilateral triangle share a side of length 4. How far is the center of mass of the square from the center of mass of the two shapes together?
28. The term for an array of one-dimensional trees of unit height planted at the lattice points in one quadrant of a square lattice is named after a famous Greek mathematician. In which episode of Numb3rs does Charlie use this term?
29. Let S be the set of 6-digit numbers with the following properties. All elements of S have the same number of distinct prime factors and any 6-digit number not in S has fewer distinct prime factors than any element of S . How many elements does S have?
30. How many primes less than 100,000,000 are palindromes when expressed in base 23?
31. I am going to pick a number from one of two normal distributions. Both normal distributions have a mean of zero, but the first has a standard deviation of 1 while the second has a standard deviation of 2. With probability $\frac{3}{4}$ I am going to select a number from the first distribution. If I tell you that the number I just selected was 1, how likely, to 3 significant figures, is it that I selected from the first distribution?
32. What is the area of the intersection of the polar graphs of $r = \sin \theta$ and $r = \cos \theta$?
33. Find all Fahrenheit temperatures that are 10 higher or lower than their Celsius equivalents.
34. How many stitches does a regulation Major League Baseball have?
35. I flip 4 fair coins and roll 12 fair six-sided dice. What is the probability, to three significant digits, that I get more heads than fives? What is the probability, to three significant digits, that I get more fives than heads?
36. Let $f_1 = f_2 = 1$ and $f_{n+1} = f_n + f_{n-1}$. Find $f_1 + f_3 + f_5 + \dots + f_i$ for odd i .

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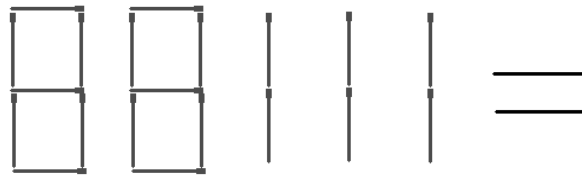
37. Find $\frac{1}{2} - \frac{3}{4} + \frac{5}{6} - \frac{7}{8} + \frac{9}{10} - \dots$

38. What is the 5th positive triangular number that is also a perfect square?

39. How many ways are there to assign a different base 10 digit to each letter of the eight letters of MEOW+MEOW = KITTY that satisfies the equality?

40. What seed is protected by US Patent 5373560?

41. Move three of the matchsticks from the left to the right side of the equal sign to make the equality true:



42. Identify the following sequence, which is the first several terms of an infinite sequence:

