Answer choice (E) NOTA means "none of these answers."

For questions 1–3, let $\vec{u} = 9i + 4j - \sqrt{3}k$ and $\vec{v} = -3i + 3j + 4\cos\frac{5\pi}{6}k$.

1. Evaluate: $||\vec{u}||$

   (A) 10 (B) $\sqrt{106}$ (C) 100 (D) 106 (E) NOTA

2. Evaluate: $\vec{u} \cdot \vec{v}$

   (A) -21 (B) $-15 - 2\sqrt{3}$ (C) $-15 + 2\sqrt{3}$ (D) -9 (E) NOTA

3. Evaluate: $\vec{u} \times \vec{v}$

   (A) $(8 + 3\sqrt{3})i + (3\sqrt{3} - 18)j + 39k$ (B) $-5\sqrt{3}i - 21\sqrt{3}j + 39k$

   (C) $-5\sqrt{3}i + 15\sqrt{3}j + 39k$ (D) $-5\sqrt{3}i + 21\sqrt{3}j + 39k$

   (E) NOTA

4. Which of the following best describes the 3-dimensional graph of $\pi x^2 + ey^2 + (\frac{1+\sqrt{5}}{2})z^2 = 100$?

   (A) Ellipsoid (B) Elliptic cone

   (C) Elliptic paraboloid (D) Hyperboloid

   (E) NOTA

For questions 5–7, let $f(x, y, z) = x^3 - 2x\cos y + xye^{z^2}$.

5. Evaluate $\frac{\partial f}{\partial x} - \frac{\partial f}{\partial z}$ at the point $(4, \pi, -1)$.

   (A) $50 + \frac{9\pi}{e}$ (B) $50 - 7\pi e$ (C) $50 + \pi e$ (D) $50 + 9\pi e$ (E) NOTA

6. Evaluate $\frac{\partial f^2}{\partial y \partial x} - f_{xz}$ at $(4, \pi, -1)$.

   (A) $(1 - 2\pi)e$ (B) $(1 + 2\pi)e$ (C) $\dfrac{1 - 2\pi}{e}$ (D) $\dfrac{1 + 2\pi}{e}$ (E) NOTA

7. What is the sum of all values of x for which $f(x, 0, \pi) = f(0, \pi, \pi)$?

   (A) -2 (B) 0 (C) 2 (D) $2\sqrt{2}$ (E) NOTA

For questions 8–9, let $w = 2xy$, where $x = s^3 + t^3$ and $y = \frac{s}{t^4}$.

8. Which of the following is equivalent to $\frac{\partial w}{\partial s} - \frac{\partial w}{\partial t}$?

   (A) $\frac{\partial w}{\partial x}\frac{\partial x}{\partial s} - \frac{\partial w}{\partial y}\frac{\partial y}{\partial t}$ (B) $\frac{\partial w}{\partial y}\frac{\partial y}{\partial s} - \frac{\partial w}{\partial x}\frac{\partial x}{\partial t}$

   (C) $\frac{\partial w}{\partial x}(\frac{\partial x}{\partial s} - \frac{\partial x}{\partial t}) - \frac{\partial w}{\partial y}(\frac{\partial y}{\partial s} - \frac{\partial y}{\partial t})$ (D) $\frac{\partial w}{\partial x}(\frac{\partial x}{\partial s} - \frac{\partial x}{\partial t}) - \frac{\partial w}{\partial y}(\frac{\partial y}{\partial t} - \frac{\partial y}{\partial s})$

   (E) NOTA

9. Find the value of $\frac{\partial w}{\partial s} - \frac{\partial w}{\partial t}$ when $s = -1$ and $t = 2$.

(A) $-\frac{1}{4}$ (B) $\frac{1}{4}$ (C) $\frac{3}{4}$ (D) $\frac{5}{4}$ (E) NOTA

10. Which of the following vectors represents the direction of minimum increase of $f(x, y, z) = x^3 - \ln y + \frac{1}{z}$ at the point $(8, \frac{1}{4}, \frac{1}{16})$?

(A) $144i - 3j - 192k$ (B) $-144i + 3j + 192k$

(C) $192i + 4j - 256k$ (D) $-192i - 4j + 256k$

(E) NOTA

11. A rectangular box in $\mathbb{R}^3$ has its base on the positive xy-plane with one vertex at the origin. The opposite vertex lies in the positive octant on the plane $5x + 2y + 3z = 30$. Find the maximum volume of such a box.

(A) 0 (B) $\frac{50}{3}$ (C) $\frac{100}{3}$ (D) 100 (E) NOTA

12. Find the minimum value of $f(x, y, z) = x^2 + 2y^2 + 3z^2$ subject to the constraint $-3x + 2y - 4z = 49$.

(A) -147 (B) -144 (C) 144 (D) 147 (E) NOTA

13. Which of the following is equivalent to $\int_0^2 \int_{x^2}^{2x} x \sin y dy dx$?

(A) $\int_{x^2}^{2x} \int_0^2 x \sin y dx dy$ (B) $\int_0^2 \int_{y^2}^{2y} x \sin y dx dy$

(C) $\int_0^4 \int_{2y}^{\sqrt{y}} x \sin y dx dy$ (D) $\int_0^4 \int_{\frac{1}{2}y}^{\sqrt{y}} x \sin y dx dy$

(E) NOTA

14. Which of the following is equivalent to $\int_{-2}^{1} \int_{x+2}^{4-x^2} dy dx$?

(A) $\int_{x+2}^{4-x^2} \int_{-2}^{1} dx dy$

(B) $\int_{-2}^{0} \int_{\sqrt{4-y}}^{y-2} dx dy + \int_{0}^{1} \int_{-\sqrt{4-y}}^{\sqrt{4-y}} dx dy$

(C) $\int_{0}^{3} \int_{\sqrt{4-y}}^{y-2} dx dy + \int_{3}^{4} \int_{-\sqrt{4-y}}^{\sqrt{4-y}} dx dy$

(D) $\int_{0}^{3} \int_{y-2}^{\sqrt{4-y}} dx dy + \int_{3}^{4} \int_{-\sqrt{4-y}}^{\sqrt{4-y}} dx dy$

(E) NOTA

15. Evaluate: $\int_0^1 \int_y^1 \sin x^2 dx dy$

(A) $\frac{1}{2}(\cos 1 - 1)$ (B) $\frac{1}{2}(1 - \cos 1)$ (C) $-\frac{1}{2}\cos 1$ (D) $\frac{1}{2}\cos 1$ (E) NOTA

16. Find the volume of the solid region in $\mathbb{R}^3$ bounded by the surface $z = 4 - y^2$, the xy-plane, and the planes $y = x$, $x = 0$, and $y = 2$.

(A) 2 (B) 4 (C) 8 (D) 16 (E) NOTA

17. The region of integration in the polar plane represented by $\int_0^{2\pi} \int_3^6 r dr d\theta$ is best described as

(A) a circle (B) an ellipse (C) a semicircle (D) an annulus (E) NOTA

18. Evaluate: $\int_0^2 \int_y^{\sqrt{8-y^2}} \sqrt{x^2+y^2} dx dy$

(A) $\frac{2\pi}{3}$ (B) π (C) $\frac{4\pi\sqrt{2}}{3}$ (D) 4π (E) NOTA

19. Evaluate: $\int_{-3}^4 \int_{-4}^5 \int_{-5}^6 xyz dx dy dz$

(A) $\frac{1}{8}$ (B) 15 (C) $\frac{315}{8}$ (D) $\frac{693}{8}$ (E) NOTA

20. Find the directional derivative of $f(x, y) = x^2 y + y \sin(xy^2)$ at the point $(\pi, 1)$ in the direction $\vec{v} = 3i + 4j$.

(A) $\frac{1}{5}(2 + 6\pi + (4 + 6\pi)\pi^2 \cos(\pi^2) + 4\sin(\pi^2))$

(B) $\frac{1}{5}(4 + 6\pi + (4 + 6\pi)\pi^2 \cos(\pi^2) + 4\sin(\pi^2))$

(C) $\frac{1}{5}(4 + 6\pi + (8 + 6\pi)\pi^2 \cos(\pi^2) + 4\sin(\pi^2))$

(D) $\frac{1}{5}(4 + 6\pi + (8 + 3\pi)\pi^2 \cos(\pi^2) + 4\sin(\pi^2))$

(E) NOTA

21. Find the curvature of $y = 4x - 3x^3$ at $x = \frac{2}{3}$.

(A) $\frac{1}{12}$ (B) $\frac{1}{9}$ (C) 9 (D) 12 (E) NOTA

22. How many of the following correctly describe the solution set of $\vec{u} \cdot \langle 9, 26 \rangle = 0$?

I. All two-dimensional vectors $\vec{u}$ that make an angle of $0°$ with $\langle 9, 26 \rangle$

II. All vectors $\vec{u} = \langle u_1, u_2 \rangle$ such that $u_2 = \frac{9}{26} u_1$

III. All two-dimensional vectors $\vec{u}$ parallel to $\langle 9, 26 \rangle$

IV. All two-dimensional vectors $\vec{u}$ orthogonal to $\langle 9, 26 \rangle$

(A) 0 (B) 1 (C) 2 (D) 3 (E) NOTA

23. Find the distance between the planes $-3x + 6y + 7z = 1$ and $6x - 12y - 14z = 25$.

(A) $\frac{23\sqrt{94}}{188}$ (B) $\frac{27\sqrt{94}}{188}$ (C) $\frac{23\sqrt{94}}{94}$ (D) $\frac{27\sqrt{94}}{94}$ (E) NOTA

24. Which of the following lies on the same line as $(1, -2, 3)$ and $(2, 1, 0)$?

(A) $(3, 4, 3)$ (B) $(3, -4, -3)$ (C) $(4, -7, 6)$ (D) $(4, 7, -6)$ (E) NOTA

25. Find a unit vector in the direction of $3i - 4j + 5k$.

(A) $\frac{\sqrt{3}}{2}i - \frac{2\sqrt{3}}{3}j + \frac{5\sqrt{3}}{6}k$ (B) $\frac{\sqrt{3}}{2}i + \frac{2\sqrt{3}}{3}j - \frac{5\sqrt{3}}{6}k$

(C) $\frac{3\sqrt{2}}{10}i + \frac{2\sqrt{2}}{5}j - \frac{\sqrt{2}}{2}k$ (D) $\frac{3\sqrt{2}}{10}i - \frac{2\sqrt{2}}{5}j + \frac{\sqrt{2}}{2}k$

(E) NOTA

For questions 26–27, let $x = t^3 - 4e^t$ and $y = \ln t - t^4$.

26. Evaluate $\frac{dy}{dx}$ at $t = 1$.

(A) $\frac{4e-3}{3}$ (B) $\frac{3-4e}{3}$ (C) $\frac{3}{4e-3}$ (D) $\frac{3}{3-4e}$ (E) NOTA

27. Evaluate $\frac{d^2y}{dx^2}$ at $t = 1$.

(A) $\frac{40e-21}{(3-4e)^2}$ (B) $\frac{21-40e}{(3-4e)^2}$ (C) $\frac{40e-21}{(3-4e)^3}$ (D) $\frac{21-40e}{(3-4e)^3}$ (E) NOTA

28. Suppose $\vec{r}(t) = 2ti + tj + \frac{1}{4}t^2k$ represents the location of a particle in space at time t. Find the speed of the particle at time 4.

(A) $2i + j + k$ (B) $\frac{1}{2}k$ (C) $8i + 4j + 4k$ (D) 3 (E) NOTA

29. Convert from cylindrical to rectangular coordinates: $(4, \frac{7\pi}{6}, 3)$

(A) $(-2, -2\sqrt{3}, 3)$ (B) $(-2\sqrt{3}, -2, 3)$ (C) $(4, -\frac{3\sqrt{3}}{2}, -\frac{3}{2})$ (D) $(4, -\frac{3}{2}, -\frac{3\sqrt{3}}{2})$ (E) NOTA

30. For $\vec{u}, \vec{v}, \vec{w} \in \mathbb{R}^3$, which of the following is equivalent to $\vec{u} \cdot (\vec{v} \times \vec{w})$?

(A) $\vec{v} \cdot (\vec{u} \times \vec{w})$ (B) $\vec{w} \cdot (\vec{v} \times \vec{u})$ (C) $\vec{u} \cdot (\vec{w} \times \vec{v})$ (D) $\vec{v} \cdot (\vec{w} \times \vec{u})$ (E) NOTA