# LIMITS & DERIVATIVES

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Directions: Choose the correst answer. If none of the listed answers are correct, enter "E" on your answer sheet for "None of these"

1. $\lim_{x\to\infty}\frac{7x^2}{x^5+7}$

A. 1 B. 7 C. $\infty$ D. 0

2. Find $\lim_{t\to 4}[t(1+\cos 2\pi t)+x^2-\cos^2 2\pi t-2\pi\sin 2\pi t]$

A. $x^2+7$ B. $x^2+8+2\pi$ C. $x^2+9$ D. 23

3. $x(t)=\cos^2(t^2)+\ln t,\quad y(t)=\sin^2 t+4t^2+2.$ Find $\frac{dy}{dx}$

A. $\frac{1+8t}{\cos t\sin t+1}$ B. $\frac{t\sin 2t+8t^2}{-2t^2\sin(2t^2)+1}$ C. $\frac{t\sin 2t+8t^2}{t\sin(4t^2)+1}$ D. $\frac{\sin 2t+8t}{\sin(2t^2)+1}$

4. $y=\sqrt{25-x^2}$ Find $\frac{dx}{dy}$

A. $\frac{-x}{y}$ B. $\frac{-\sqrt{25-x^2}}{x}$ C. $-x$ D. $\frac{-x\sqrt{25-x^2}}{25-x^2}$

5. A rope is fastened to the ground at a distance of eight feet from a vertical wall. A monkey climbs the wall holding onto the loose end of the rope. If the monkey holds the rope taut and climbs at a rate of four feet per second, at what rate is the length of the rope ( between the monkey's hand and where it is fastened to the ground) increasing when the monkey is six feet up the wall.

A. .24 ft/s B. 2.4 ft/s C. 1.2 ft/s .4 ft/s

6. $y=x^3-3x^2+5$ . Find the interval(s) for which the slope is increasing.

A. $x>0$ B. $x>1$ C. $0<x<2$ D. $x<0$ or $x>2$

7. If the position of a particle is given by the vector $\mathbf{r} = (3t^2 + 2t)\mathbf{i} + (\cos t + 4t^3)\mathbf{j} + (e^{2t} + 4)\mathbf{k}$ find the velocity.

A. $(6t+2)\mathbf{i} + (12t^2 - \sin t)\mathbf{j} + (2e^{2t})\mathbf{k}$

B. $(t^3 + t^2)\mathbf{i} + (\sin t + t^4)\mathbf{j} + \left(\frac{1}{2}e^{2t} + 4t\right)\mathbf{k}$

C. $6\mathbf{i} + (12t + \cos t)\mathbf{j} + 2\mathbf{k}$

D. $(3t+2)\mathbf{i} + (\sin t + 4t)\mathbf{j} + (2e^{2t})\mathbf{k}$

8. $f(x) = \dfrac{1+\sin x}{1-\cos x}$, find $f'\left(\dfrac{\pi}{2}\right)$

A. 3 B. -1 C. -2 D. $\frac{1}{2}$

9. Suppose $f(x)$ has a derivative at $x = c$, and $g(x)$ has a derivative at $x = c$. Then

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right)$$

A. equals c B. has a derivative at $c$ C. has a critical point at $c$ D. equals 1

10. A point P is moving along the part of the curve $x = y^2$ which is in the first quadrant in such a way that its $x$ coordinate is increasing at a rate of 5 units/s. A rectangle with a vertex at the origin, and two sides on the coordinate axes, has P as its other vertex. At what rate is the area of this rectangle changing when $x = 9$?

A. 67.5 units/s B. 3.5 units/s 1215 units/s 22.5 units/s

11. In a newly incorporated city, it is estimated that the voting population (in thousands) will be $N(t) = 30 + 12t^2 - t^3 \quad 0 \le t \le 8$, where $t$ is measured in years. When will the rate increase be the most rapid?

A. $t = 0$ B. t = 8 C. $t = 4$ D. $t = 6$

12. $f(x) = \sqrt{x^2+1} \quad g(x) = x - 1$, find $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)}$

A. 1 B. 0 C. $\infty$ D. $\frac{1}{2}$

13. $\lim_{x\to 0}\dfrac{\sqrt{x+4}-2}{x}=$

A. 4 B. 0 C. $\infty$ D. $\dfrac{1}{4}$

14. $\lim_{x\to\infty}\dfrac{\frac{1}{4}\sin 3x+e^{-x}}{x}$

A. 1 B. 0 C. $\infty$ D. $\dfrac{3}{4}$

15. The Maclaurin Series of a function is given by the series

$$f(x)=f(0)+f'(0)x+f''(0)\frac{x^2}{2!}+f'''(0)\frac{x^3}{3!}\cdots$$

If $f(x)=\cos x$, find the coefficient for $x^6$

A. $-\dfrac{1}{6!}$ B. $-\dfrac{1}{6}$ C. 0 D. $\dfrac{1}{6!}$

16. The marketing research department of Fake Tattoos Unlimited test marketed their temporary tattoos in novelty shops throughout the state and found that the relation between the price of the tattoos and the weekly demand is $p=12-2\ln x \quad 0<x<90$, where $x$ = the number of tattoos sold weekly, and $p$ = price of one package of tattoos. If each package costs \$3.00 to produce, how should they price them to maximize profit?

A. \$5.50 B. \$5.00 \$10.00 \$3.50

17 $\lim_{n\to\infty}\dfrac{3}{n}\left(\dfrac{5}{n^2}\cdot\dfrac{n(n+1)(2n+1)}{6}-\dfrac{1}{n}\cdot\dfrac{n(n+1)}{2}\right)$

A. 0 B. $\infty$ C. $\dfrac{5}{2}$ D. $\dfrac{7}{2}$

18. $f(x)=\int_1^{x^2}\dfrac{3}{t}dt$ find $f'(x)$

A. $\dfrac{6}{x}$ B. $\dfrac{3}{x^2}$ C. $\dfrac{2}{3x^2}$ D. $\dfrac{3}{x^2}-1$

19. $f(x)$ is a curve that passes through $(-1,2)$ and $(4,6)$, and whose slope at any point is proportional to its $x$ coordinate. $f(x)$ is a

A. parabola B. circle C. line D. ellipse

20. $\lim_{x\to\infty}\frac{\ln x}{x^5}$

A. $\frac{1}{5}$ B. $\infty$ C. $e$ D. 0

21. $\frac{y}{\sqrt{x}-y}=2+x^2$ find $\frac{dy}{dx}$

A. $\frac{3x+2yx^{\frac{1}{2}}+y^2+2yx^{-\frac{1}{2}}}{x^{\frac{1}{2}}-2y}$ B. $\frac{2+5x^2-4x^{\frac{3}{2}}y}{2x^{\frac{1}{2}}\left(x^2+3\right)}$

C. $\frac{2+x^{\frac{3}{2}}+4x^3-4x^2y}{2x\left(x^2+3\right)}$ D. $\frac{1+5x^2}{2\sqrt{x}}$

22. $\lim_{x\to 0}\left|\frac{x+2}{x}\right|$

A. 1 B. 0 C. $\infty$ D. 2

23. $\lim_{x\to\frac{1}{2}}\frac{\sin^{-1}x-\frac{\pi}{6}}{x-\frac{1}{2}}$

A. $\frac{4}{3}$ B. $\frac{2\sqrt{3}}{3}$ C. 1.83 D. 4.35

24. $f(x)=\cot^{-1}(\ln(\sin 2x))$, find $f'\left(\frac{\pi}{4}\right)$

A. 0 B. -1 C. $\infty$ D. does not exist

25. $\lim_{t\to 0}\frac{1}{x}\cdot\ln\left[(1+tx)^{\frac{1}{t}}\right]$

A. 1 B. 0 C. $\infty$ D. $\frac{1}{x}$

26. A fisherman hooks a fish which is traveling in an east-west direction along a line 30 feet north of the fisherman. If the fishing line leaves the reel at a rate of six feet/sec, when the fish is 50 feet from the fisherman, how fast is the fish traveling?

A. 6 ft/s B. 4.8 ft/s C. 3.6 ft/s D. 7.5 ft/s

27. $\lim_{x\to 0}\frac{e^x}{x\sin x}$

A. 1 B. 0 C. $\infty$ D. $e$

28. Find the slope of the tangent line at of the graph $y=x^4+x^3-3x^2$ where the rate of change of the slope is zero.

A. $-\frac{7}{4},5$ B. $\frac{1}{2},-1$ C. $0,\frac{-3\pm\sqrt{105}}{8}$ D. $1,-3$

29. $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{x\tan x}\right)$

A. 0 B. -1 C. $\infty$ D. $\frac{1}{3}$

30. Find $y''$ when $x^3y+xy^3=2$, y = 1 and $x=1$

A. 0 B. 1 C. -1 D. 2