Solutions - Limits and Derivatives Topic Test 1992-1993 State Convention

1. $\lim_{x\to 1} \frac{1-x}{\sqrt{12-3x}-3} = \frac{0}{0} \to \lim_{x\to 1} \frac{-1}{\frac{-3}{2\sqrt{12-3x}}} = 2$ **C.**

2. $y = 2\cos\left(\frac{x}{2}\right)$ $y' = -\sin\left(\frac{x}{2}\right)y' = -\sin\left(-\frac{\pi}{2}\right) = -1$ **E.**

3. $\lim_{x\to 1} \frac{x^3-x^2+2x-2}{x-1} = \lim_{x\to 1} \frac{(x-1)(x^2+2)}{(x-1)} = 3$ **D.**

4. $f(x) = (x^3+2)^2$ $f'(x) = 6x^2(x^3+2)$

$\to f'(-1) = 6(-1+2) = 6$ **D.**

5. $f(g(x)) = x$ $f'(g(x))g'(x) = 1$ $g'(x) = \frac{1}{f'(g(x))}$

Find x such that $f(x) = -49 \to x = -2$.

Find $f'(-2)$. $f'(-2) = 7(-2)^6 + 20(-2)^3 + 4(-2)$

$f'(-2) = 280$ $g'(x) = \frac{1}{f'(g(x))} = \frac{1}{280}$ **B.**

6. $\lim_{n\to\infty} \sqrt{n^2+1} - n = \lim_{n\to\infty} \frac{(\sqrt{n^2+1}-n)(\sqrt{n^2+1}+n)}{\sqrt{n^2+1}+n}$

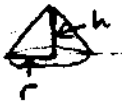
$\to \lim_{n\to\infty} \frac{1}{\sqrt{n^2+1}+n} = \frac{1}{\infty} = 0$ **A.**

7. $\lim_{t\to x} \frac{t^{11}-x^{11}}{t-x} = \frac{0}{0} \to \lim_{t\to x} \frac{11t^{10}}{1} = 11x^{10}$ **D.**

8. $y = \frac{ax-1}{a^2}(e^{ax})$ $y = \frac{1}{a^2}(axe^{ax} - e^{ax})$

$y' = \frac{1}{a^2}(ax(ae^{ax}) + ae^{ax}) - \frac{1}{a^2}(ae^{ax})$

$y' = xe^{ax} + \frac{1}{a}e^{ax} - \frac{1}{a}e^{ax} = xe^{ax}$ **B.**

9. 

$\frac{dV}{dt} = 9$ $h = \frac{2}{3}r$ $\frac{dh}{dt} = \frac{2}{3}\frac{dr}{dt}$

$V = \frac{1}{3}\pi r^2 h \to V = \frac{2}{9}\pi r^3$ $\frac{dV}{dt} = \pi r^2\left(\frac{2}{3}\frac{dr}{dt}\right)$

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$ $\to \frac{9}{\pi(6^2)} = \frac{1}{4\pi} = \frac{dh}{dt}$ **B.**

10. $\lim_{y\to\infty}\left(\lim_{x\to\infty} \frac{Ax+By}{Cx+Dy}\right) = \frac{A}{C}$ $\lim_{x\to\infty}\lim_{y\to\infty}\left(\frac{Ax+By}{Cx+Dy}\right) = \frac{B}{D}$

$\frac{A}{C} - \frac{B}{D} = \frac{AD-BC}{AC}$ **D.**

11. $0 < x+1 < \delta$ $|f(x)-L| < \epsilon$

$|(5x+3)-(-2)| < .0025 \to |5x+5| < .0025$

$5|x+1| < .0025$ $|x+1| < .0005 = \delta$ **D.**

12. $y = x^2 + y^2$ $y' = 2x + 2yy'$

$y' = \frac{2x}{1-2y}$ $y' = \frac{2(0)}{1-2(1)} = 0$ **A.**

13. $\lim_{x\to 10^+} \sin \pi[\![x]\!] = \sin 10\pi = 0$

$\lim_{x\to 10^-} \sin \pi[\![x]\!] = \sin 9\pi = 0$

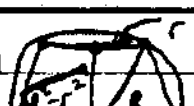
$\therefore \lim_{x\to 10} \sin \pi[\![x]\!] = 0$ **B.**

14. $A = \pi(r_2^2 - r_1^2)$ $A' = 2\pi r_2 r_2' - 2\pi r_1 r_1'$

$A' = 2\pi(6(.01) - 4(.02)) = -.04\pi$ **B.**

15. $\lim_{x\to 0} \frac{\sin(Ax-B)+\sin(Ax+B)}{x} = \frac{\sin -B + \sin B}{0} = \frac{0}{0}$

$\to \lim_{x\to 0} \frac{A\cos(Ax-B) + A\cos(Ax+B)}{1} = 2A\cos B$ **C.**

16. 

$A = \frac{1}{2}\left(\sqrt{R^2-r^2}\right)(2r+2$

$A = r\sqrt{R^2-r^2} + R\sqrt{R^2-r^2}$

$A' = r\frac{(-2r)}{2\sqrt{R^2-r^2}} + \sqrt{R^2-r^2} + \frac{R(-2r)}{2\sqrt{R^2-r^2}} \to A' = 0$

$0 = \frac{-r^2-Rr}{\sqrt{R^2-r^2}} + \sqrt{R^2-r^2} \to \frac{r^2+Rr}{\sqrt{R^2-r^2}} = \sqrt{R^2-r^2}$

$r^2 + Rr = R^2 - r^2 \to 2r^2 + Rr - R^2 = 0$

$(2r-R)(r+R) = 0$ $2r = R$ $r = \frac{R}{2}$

$A = (r+R)\sqrt{R^2-r^2} = \left(\frac{3r}{2}\right)\left(\sqrt{\frac{3r^2}{4}}\right) = \frac{3\sqrt{3}r^2}{4}$

17. $P = \lim_{t\to\infty} P_0\left(1+\frac{1}{t}\right)^{ty} = P_0 e^y$

$\to 3P_0 = P_0 e^y$ $\ln 3 = y$ **B.**

18. $\lim_{x\to a} \frac{(x-a)(x^2+ax+a^2)}{(x-a)(x+a)(x^2+a^2)} = \frac{3a^2}{4a^3} = \frac{3}{4a}$ **C.**

19. $y = \lim_{a\to 0} (2ax+1)^{\frac{1}{a}} \to \ln y = \lim_{a\to 0} \frac{1}{a}\ln(2ax+1)$

$\ln y = \lim_{a\to 0} \frac{\frac{2x}{2ax+1}}{1} = 2x \quad y = e^{2x}$ **C.**

20. $f(x) = \sin(\ln(x))^3$

$f'(x) = 3[\ln x]^2\left(\frac{1}{x}\right)\cos(\ln(x))^3$ **C.**

21. $f(x) = (\cos^2 x - \sin^2 x)^2 - (2\sin x\cos x)^2$

$f(x) = \cos^2 2x - \sin^2 2x = \cos 4x$

$f'(x) = -4\sin 4x$ **D.**

22. $f_n'(x)$ is always 3 for all x

$h(x) = f_1(f_2(f_3\cdots f_n(x)) \to h'(x) = f_1'(f_2\cdots)f_2'(f_3\cdots$

$\cdots f_n'(x) \quad h'(x) = 3^n$ **A.**

23. $\lim_{x\to\infty} \frac{x+\sin x}{2x+5} = \lim_{x\to\infty} \frac{1+\frac{\sin x}{x}}{2+\frac{5}{x}} = \frac{1}{2}$ **A.**

24. $\lim_{n\to\infty} n f\left(\frac{1}{n}\right) = \lim_{n\to\infty} \frac{f\left(\frac{1}{n}\right)}{\frac{1}{n}} = \frac{0}{0}$

$\lim_{n\to\infty} \frac{f'\left(\frac{1}{n}\right)\left(\frac{1}{n^2}\right)}{-\frac{1}{n^2}} = f'(0)$ **B.**

25. $dy/dt = 6\sin 2t \qquad dx/dt = 8\cos 2t$

$dy/dx = -\frac{4}{3}\cot 2t \qquad \frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}$

$\to \frac{d^2y}{dx^2} = \frac{\frac{8}{3}\csc^2 2t}{-6\sin 2t}$ @ $t = \frac{\pi}{4}$ $\frac{d^2y}{dx^2} = -\frac{4}{9}$ **B.**

26. $f(x) = \ln(1+x) \quad f'(x) = \frac{1}{1+x} \quad f^2(x) = \frac{-1}{(1+x)^2}$

$f^3(x) = \frac{2}{(1+x)^3} \quad f^4(x) = \frac{-6}{(1+x)^4} \quad f^5(x) = \frac{24}{(1+x)^5}$

$\ln(1+x) = f(0) + f'(0)x + \frac{f^2(0)x^2}{2} + \frac{f^3(0)x^3}{6}$ continued

$\to \frac{f^4(0)x^4}{24} + \frac{f^5(0)x^5}{120}$

$\ln(1+x) = 0 + x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}$

at $x=1$ $\ln(1+x) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} = \frac{47}{60}$ **A.**

27. $3(x^2+y^2)^2 = 100xy$

$6(x^2+y^2)(2x+2yy') = 100xy' + 100y$

use (3,1) $6(10)(6+2y') = 100(3)y' + 100$

$360 + 120y' = 300y' + 100$

$260 = 180y'$

$\frac{13}{9} = y'$ **A.**

28. $\frac{d}{dt}\int_t^{t^3} \frac{(x^3-1)^6}{x^2+1}\,dx$

$3t^2\left(\frac{(t^9-1)^6}{t^6+1}\right) - \left(\frac{(t^3-1)^6}{t^2+1}\right)$ at $t=-1$

$\frac{3\cdot 64}{2} - \frac{64}{2} = 64$ **C.**

29. $f(x) = x(\text{Arcsin } x)^2 - 2x + 2\sqrt{1-x^2}\,\text{Arcsin } x$

$f'(x) = \frac{x(2\text{Arcsin } x)}{\sqrt{1-x^2}} + (\text{Arcsin } x)^2 - 2 + \frac{2\sqrt{1-x^2}}{\sqrt{1-x^2}}$

$+ \frac{2\text{Arcsin } x(-2x)}{2\sqrt{1-x^2}}$

$f'(x) = (\text{Arcsin } x)^2$

$f'(1) = \frac{\pi^2}{4}$ **B.**

30. $y = ae^{bx} + ce^{dx} \quad y' = abe^{bx} + cde^{dx}$

$y'' = ab^2e^{bx} + cd^2e^{dx}$ | When $y=5, x=0$: at c:

$2y'' - 5y' - 12y = 0$

$2ab^2e^{bx} + 2cd^2e^{dx} - 5abe^{bx} - 5cde^{dx} - 12ae^{bx} - 12ce^{d}$

$ae^{bx}(2b^2 - 5b - 12) + ce^{dx}(2d^2 - 5d - 12) = 0$

$ae^{bx}(2b+3)(b-4) + ce^{dx}(2d+3)(d-4) = 0$

$b = -\frac{3}{2}$ or $4 \qquad d = 4$ or $-\frac{3}{2}$

$b + d = \frac{5}{2}$

$a + c = 5$

$a + b + c + d = 7.5$ ~~B.~~

changed to D