Limits and Derivatives

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Grenelefe, FL

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1. Evaluate: $\lim_{x \to -\infty} (\sqrt{x^2 + 5x} + x)$

   a. $-\infty$ b. $-5/2$
   c. 0 d. 5/2 e. nota

2. If $f(x) = \sqrt[3]{x}$, determine $\lim_{x \to 0} \frac{f(x+h) - f(x)}{h}$.

   a. 0 b. $\frac{3\sqrt[3]{x}}{2}$
   c. ∞ d. does not exist e. nota

3. Evaluate: $\lim_{x \to 0} \frac{x(\cos x - 1)}{\sin x - x}$

   a. 0 b. 1
   c. 2 d. 3 e. nota

4. If $y = f(x^2)$ and $f'(x) = \sqrt{5x - 1}$, then $\frac{dy}{dx} = ?$

   a. $2x\sqrt{5x^2 - 1}$ b. $\sqrt{5x - 1}$
   c. $2x\sqrt{5x - 1}$ d. $\sqrt{5x - 1}\,/2x$ e. nota

5. $\lim_{x \to +\infty} \frac{2x - \cos x}{\sin x + 3x}$

   a. 0 b. 1/2
   c. 2/3 d. 2 e. nota

6. If $y = \sqrt{x^3 + 1}$, find $D_y x$.

   a. $\frac{3x^2}{2\sqrt{x^3 + 1}}$ b. $\frac{1}{2\sqrt{x^3 + 1}}$

   c. $\frac{3x^2\sqrt{x^3 + 1}}{2}$ d. $\frac{2\sqrt{x^3 + 1}}{3x^2}$ e. nota

7. Find the value of the partial derivative of x with respect to z for the equation $3x^2 - 2y^2 - 4z^2 + 6x + 9y + 8z - 9 = 0$ at the point (1, 0, 2).

a. –3/2 b. –2/3
c. 2/3 d. 3/2 e. nota

8. Let f and g be differentiable functions such that

$f(1) = 2$ $f'(1) = 3$ $f'(2) = -4$
$g(1) = 2$ $g'(1) = -3$ $g'(2) = 5$

If $h(x) = f(g(x))$, then $h'(1) =$

a. –9 b. –4
c. 0 d. 12 e. nota

9. If $f'(x) = h(x)$ and $g(x) = x^3$, then $\frac{d}{dx} f(g(x)) =$

a. $3x^2 h(x^3)$ b. $3x^2 h(x)$
c. $h'(x)$ d. $9x^4 h(x^3)$ e. nota

10. Given: $\ln(xy) = x$. Find $\frac{dy}{dx}$.

a. $\frac{x-1}{x}$ b. $\frac{(x-1)e^x}{x^2}$
c. $\frac{y+e^x}{x}$ d. xe^x e. nota

11. Given: $y = \tan(\arccos x)$ Find y'.

a. $\frac{-1}{\sqrt{1-x^2}}$ b. $\frac{-1}{x^2\sqrt{x^2-1}}$
c. $\frac{-1}{x^2\sqrt{1-x^2}}$ d. $\frac{1}{x^2\sqrt{1-x^2}}$ e. nota

12. Evaluate: $\lim_{n \to \infty} \frac{1+2+3+4+\cdots+n}{(n-1)^2}$

a. 1/2 b. 1
c. 2 d. ∞ e. nota

13. Given x and y are differentiable functions of t and $\frac{dy}{dx} = t^3 + 3t$ and $\frac{d^2y}{dx^2} = t^2 + 1$, find $\frac{dx}{dt}$.

a. $\frac{t^3 + 3t}{t^2 + 1}$ b. $\frac{3t^2 + 3}{2t}$

c. $\frac{t^2 + 1}{t^3 + 3t}$ d. 3 e. nota

14. A point moves on the x-axis in such a way that its velocity at time t ($t > 0$) is given by $v = \frac{\ln t}{t}$. At what value of t does v attain its maximum?

a. 1 b. $e^{.5}$

c. e d. $e^{1.5}$ e. nota

15. Given: $\lim_{x \to 0} \frac{A + \cos Bx}{x^2} = -4$. Find $A^2 + B^2$. [A and B are constants.]

a. 1 b. 5

c. 7 d. 9 e. nota

16. $\lim_{x \to \infty} \frac{\int_0^{2x} \sqrt{1 + t^3}\, dt}{x^{5/2}}$

a. $\frac{2\sqrt{2}}{5}$ b. $\frac{4\sqrt{2}}{5}$

c. $\frac{8\sqrt{2}}{5}$ d. $\frac{4}{5}$ e. nota

17. Let $(-2, g(-2))$ be a relative maximum for $g(x) = 2x^3 + Ax^2 + Bx - 6$ where A and B are constants. If $(-1/2, g(-1/2))$ is an inflection point, find the value of $A - B$.

a. −15 b. −9

c. 9 d. 15 e. nota

18. Given $f(x) = \frac{x^3}{3} + \frac{x^2}{2} - x + \ln|x|$. At what value of x does $x \cdot f'(x)$ have a relative maximum?

a. −2 b. −1

c. 0 d. 1/3 e. nota

19. Given: $f(x) = x^8 - 2x + 3$ and $x_0 = -1/2$. Find $\lim_{h \to 0} \frac{f'(x_0 + h) - f'(x_0)}{h}$

a. $-7/8$ b. $-33/16$
c. $7/8$ d. 56 e. nota

20. If $y = x^{\ln x}$, find $\frac{dy}{dx}$.

a. $2 \ln x \cdot x^{\ln x - 1}$ b. $\frac{x \ln x^{\ln x - 1}}{x}$
c. $\frac{2 \ln x}{x}$ d. $\frac{\ln x \cdot x^{\ln x}}{x}$ e. nota

21. Given: $f(x) = \frac{\tan x}{\sec^2 x}$ and $g(x) = \frac{e^{x^2}}{2}$. If $h(x) = f(g'(x))$, then $h'(1) = ?$

a. $3e \cos 2e$ b. $3e \cos^2 2e - 3e$
c. $3e \sin 2e$ d. $6e \cos 2e$ e. nota

22. If $y = \sqrt{\ln (x^2 + 1)}$, $(x > 0)$, find the rate of change of y^2 with respect to $\ln x$.

a. $\frac{2x}{x^2 + 1}$ b. $\frac{2}{x^2 + 1}$
c. $\frac{2x}{\ln x(x^2 + 1)}$ d. $\frac{2x^2}{x^2 + 1}$ e. nota

23. Evaluate: $\lim_{x \to 0} x^{-3} (\tan^{-1} x - x)$

a. -1 b. $-1/3$
c. $1/3$ d. 1 e. nota

24. If $a_n = \frac{(1 + \sqrt{5})^n - (1 - \sqrt{5})^n}{2^n \cdot \sqrt{5}}$, where a_n is the nth term of the Fibonacci sequence, then $\lim_{n \to \infty} \frac{a_{n+1}}{a_n} = ?$

a. $\frac{1 - \sqrt{5}}{2}$ b. 0
c. 1 d. $\frac{1 + \sqrt{5}}{2}$ e. nota

25. Given: $f(x) = \sum_{k=0}^{\infty} \frac{x^k}{k!}$. Find $\lim_{x \to 0} f'(x)$.

a. -1 b. 0

c. 1 d. ∞ e. nota

26. Given: $x = y^3 + y$. Find $\frac{d^2y}{dx^2}$ when $x = 2$.

a. $-3/104$ b. $-3/8$

c. 3/32 d. 3/16 e. nota

27. Find the slope of the normal to the graph of $y + 4 = x^2 + 2\pi \sin^2 (\frac{y}{2x})$ at $(2, \pi)$.

a. $\frac{\pi^2 - 16}{2\pi - 4}$ b. $\frac{\pi^2 - 16}{4 - 2\pi}$

c. $\frac{4 - 2\pi}{\pi^2 - 16}$ d. $\frac{\pi - 4}{16}$ e. nota

28. If $f(x) = e^{g(x)}$ and $g(x) = \int_2^x \frac{t}{1 + t^4}\, dt$, find $f'(2)$.

a. 0 b. 1/33

c. $2e^2/17$ d. does not exist e. nota

29. Evaluate: $\lim_{x \to 0} |x| \cdot \csc^2 \sqrt{2|x|}$

a. 0 b. 1/2

c. 1 d. 2 e. nota

30. If $g(x) = f^{-1}(x)$ and $f(x) = x^3 + x - 1$, find $g'(9)$.

a. 1/244 b. 1/13

c. 13 d. 244 e. nota