

⑦ Equations and Inequalities Topic Test Solutions

① $2^3 + 2a + 6 = 0$ A
 $2a = -14$
 $a = -7$

②
$$\begin{array}{r|rrrr} 7 & 1 & 0 & -9 & -28 \\ & & 7 & 49 & 280 \\ \hline & 1 & 7 & 40 & 252 \end{array} \quad X$$
 E

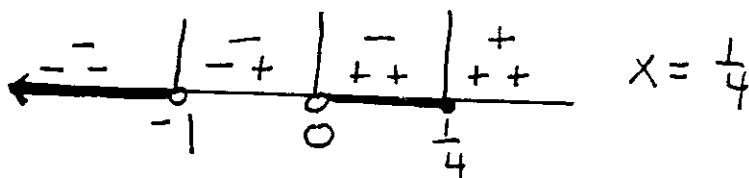
$$\begin{array}{r|rrrr} -4 & 1 & 0 & -9 & -28 \\ & & -4 & 16 & -28 \\ \hline & 1 & -4 & 7 & -56 \end{array} \quad X$$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -9 & -28 \\ & & 2 & 4 & -10 \\ \hline & 1 & 2 & -5 & -38 \end{array} \quad X$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -9 & -28 \\ & & -2 & 4 & +10 \\ \hline & 1 & -2 & -5 & 18 \end{array} \quad X$$

③ $\frac{2}{x^2} + \frac{1}{x} - 1 = 0$ D
 $2 + x - x^2 = 0$
 $x^2 - x - 2 = 0$
 $r_1 + r_2 = -(-1)$
 $= 1$

④ $\frac{5}{x+1} - \frac{1}{x} \leq 0$ C
 $\frac{5x - x - 1}{x(x+1)} \leq 0$
 $\frac{4x - 1}{x(x+1)} \leq 0$



$$\textcircled{5} \frac{1}{\frac{x-1}{1-x+1} - 1} = 2$$

$$\frac{2-x}{x-1-2+x} = 2$$

$$2-x = 4x-6$$

$$5x = 8$$

$$x = \frac{8}{5}$$

D

$$\textcircled{6} r_1 + r_2 + r_3 = -a$$

$$2+i+2-i+3 = -a$$

$$a = -7$$

$$r_1 r_2 r_3 = -c$$

$$(2+i)(2-i)3 = -c$$

$$c = -15$$

A

$$r_1 r_2 + r_1 r_3 + r_2 r_3 = b$$

$$5 + 6+3i+6-3i = b$$

$$b = 17$$

$$\textcircled{7} 15 + 3x + \cancel{x} > \cancel{x} - 2x$$

$$5x > -15$$

$$x > -3$$

C

$$\textcircled{8} (w-4)(w+5) + 2(w-2)(w+5) = (3w-1)(w-2) \quad D$$

$$w^2 + w - 20 + 2w^2 + 6w - 20 = 3w^2 - 7w + 2$$

$$14w = 42$$

$$w = 3$$

12

$$\begin{array}{r|rrrrr}
 1 & 1 & -4 & 1 & -2 & -1 \\
 & & 1 & -3 & -2 & -4 \\
 \hline
 & 1 & -3 & -2 & -4 & -5 \\
 -1 & 1 & -4 & 1 & -2 & -1 \\
 & & -1 & 5 & -6 & 8 \\
 \hline
 & 1 & -5 & 6 & -8 & 7
 \end{array}$$

$f(-1) = 7$ } at least 1 root
 $f(0) = -1$ } between 0 and (-1).
 (Exactly one)
 $f(1) = -5$
 $f(2) = -17$
 $f(3) = -25$ } 1 root
 $f(4) = 7$ } between 3+4.

no rational roots. -1 is a bound
 kinds of roots on neg. roots 2 Real Roots

pos 3 1
 neg 1 1
 complex 0 2

B

$$\begin{array}{r|rrrrr}
 4 & 1 & -4 & 1 & -2 & -1 \\
 & & 4 & 6 & 4 & 8 \\
 \hline
 & 1 & 0 & 1 & 2 & 7
 \end{array}$$

4 is a bound
 on pos. roots

13

$$|x-1| + |x+2| < 5$$

B

when $x \leq -2$
 $-x+1-x-2 < 5$
 $-2x < 6$
 $x > -3$
 when $-2 \leq x \leq 1$
 $-x+1+x+2 < 5$
 $3 < 5$ all

when $x \geq 1$
 $x-1+x+2 < 5$
 $2x < 4$
 $x < 2$

$\therefore -3 < x < 2$

$$\textcircled{18} \quad (x^2 - 3x)^2 - 3(x^2 - 3x) - 4 = 0$$

$$x^2 - 3x - 4 = 0 \quad x^2 - 3x + 1 = 0$$

$$r_1 + r_2 = 3 \quad r_1 + r_2 = 3$$

$$3 + 3 = 6$$

A

$$\textcircled{19} \quad 3x + 6 = A(x^2 + 2)^2 + (Bx + C)(x^2 + 2)(x - 1) + (Dx + E)(x - 1)$$

Let $x = 1$

$$\therefore 9 = 9A \rightarrow A = 1$$

ans D

Let $x = 0$

$$\textcircled{1} \quad 6 = 4 + C(2)(-1) + E(-1) \rightarrow 2C + E = -2$$

Let $x = 2$

$$\textcircled{2} \quad 12 = 36 + (2B + C)6 + 2D + E \rightarrow 12B + 6C + 2D + E = -24$$

Let $x = -1$

$$\textcircled{3} \quad 3 = 9 + (-B + C)3(-2) + (-D + E)(-2) \rightarrow -3B + 3C - D + E = 3$$

Let $x = -2$

$$\textcircled{4} \quad 0 = 36 + (-2B + C)6(-3) + (-2D + E)(-3) \rightarrow -12B + 6C - 2D + E = 12$$

$$\textcircled{2+4} \quad 12C + 2E = -12$$

$$\begin{cases} 6C + E = -6 \\ 2C + E = -2 \end{cases}$$

$$A + B + C + D + E =$$

$$C = -1 \quad 1 - 1 - 1 - 3 + 0 = -4$$

$$\begin{cases} 3B + 3 + D = -3 \\ 12B + 6 + 2D = -12 \end{cases}$$

$$6B = -6$$

$$B = -1$$

$$D = -3$$

$$E = 0$$

$$\textcircled{23} \quad f(2x-3) = 2(2x-3)^3 - 4(2x-3)^2 + 7(2x-3) + 1$$

$$= 16x^3 - 72x^2 + \dots + (-16x^2)$$

B

$$r_1 + r_2 + r_3 = (-1) \frac{a_1}{a_0} = \frac{(-1)(-88)}{16} \quad \begin{matrix} a_1 = -88 \\ a_0 = 16 \end{matrix}$$

$$= \frac{11}{2}$$

$$\textcircled{24} \quad \frac{1}{r^2} + \frac{1}{s^2} + \frac{1}{t^2} = \frac{s^2 t^2 + r^2 t^2 + r^2 s^2}{r^2 s^2 t^2}$$

A

$$rst = 7 \rightarrow r^2 s^2 t^2 = 49$$

$$(rst)^2 = 5$$

$$r^2 s^2 + 2r^2 s t + r^2 t^2 + 2r s^2 t + 2r t^2 + s^2 t^2 = 25$$

$$r^2 s^2 + r^2 t^2 + s^2 t^2 + 2rst(r+s+t) = 25$$

$$r^2 s^2 + r^2 t^2 + s^2 t^2 + 2(7)(6) = 25$$

$$r^2 s^2 + r^2 t^2 + s^2 t^2 = -59$$

$$\textcircled{25} \quad z^2 = 6 - 8i; \quad z = a + bi$$

$$z = \pm \left(\sqrt{\frac{r+a}{2}} - i \sqrt{\frac{r-a}{2}} \right) \quad \begin{matrix} r=10 \\ a=6 \end{matrix}$$

B

$$z = \pm \left(\sqrt{\frac{6+10}{2}} - i \sqrt{\frac{10-6}{2}} \right)$$

$$z = \pm (2\sqrt{2} - \sqrt{2}i)$$

$$a = 2\sqrt{2} \quad b = -\sqrt{2}$$

$$a^2 + b^2 = 8 + 2 = 10$$