



- 1) Simplify:  $(3 + i) + (9 - 3i) - 2(7 + 4i)$
- A)  $-2 - 10i$       B)  $-2 + 6i$       C)  $5 - 6i$       D)  $5 + 10i$       E) NOTA
- 2) Simplify:  $(3 + i)(2 - 3i)$
- A)  $3 - 7i$       B)  $9 - 7i$       C)  $6 - 3i$       D)  $6 + 11i$       E) NOTA
- 3) Simplify:  $\frac{3 + i}{2 - 3i}$
- A)  $\frac{9}{13} - \frac{7}{13}i$       B)  $\frac{3}{13} - \frac{11}{13}i$       C)  $\frac{9}{13} + \frac{7}{13}i$       D)  $\frac{3}{13} + \frac{11}{13}i$       E) NOTA
- 4) If  $(6 - 6i\sqrt{3})^5 = a + bi$  where  $a$  and  $b$  are real numbers, then find the ratio  $\frac{a}{b}$ .
- A)  $-\sqrt{3}$       B)  $-\frac{\sqrt{3}}{3}$       C)  $\frac{\sqrt{3}}{3}$       D)  $\sqrt{3}$       E) NOTA
- 5) What is the real part of  $(\sqrt{2} + 2i)(-i\sqrt{2} + 1)(2\sqrt{2} - i)$ ?
- A)  $12 + 2\sqrt{2}$       B)  $14 - \sqrt{2}$       C)  $-3\sqrt{2}$       D) 12      E) NOTA
- 6) What is the imaginary part of  $(1 + i)^2 \cdot e^{-i\pi/4}$ ?
- A)  $-2\sqrt{2}$       B)  $-\sqrt{2}$       C)  $\sqrt{2}$       D)  $2\sqrt{2}$       E) NOTA
- 7) Two complex numbers,  $z_1$  and  $z_2$  satisfy the two following properties:  
 $z_1 + z_2$  is purely imaginary, and  $z_1^2 + z_2^2$  is real.  
Both  $z_1$  and  $z_2$  have non-zero real and imaginary parts.  
For the two properties to hold, which of the following statement(s) MUST be true?
- I.)  $z_1 = \overline{z_2}$   
II.)  $z_1 - z_2$  is purely real.  
III.)  $z_1$  cannot be equal to  $z_2$   
IV.)  $|z_1| = |z_2|$
- A) I. and II. only      B) II. and IV. only      C) II., III., and IV.      D) All are true      E) NOTA
- 8) There are two complex zeros of the function  $f(z) = z^2 + 2z + 16$ ,  $z_1$  and  $z_2$ . What is the sum of the squares of  $z_1$  and  $z_2$ ?
- A)  $-28$       B)  $-15$       C) 0      D) 32      E) NOTA



- 9) Complex numbers can be thought of as vectors in the complex plane. What is the angle that the vector starting at  $3 - i$  and ending at  $-1 + 3i$  makes with the positive  $x$ -axis?
- A)  $45^\circ$                       B)  $135^\circ$                       C)  $225^\circ$                       D)  $315^\circ$                       E) NOTA
- 10) The sum  $1 + (1 - i) + (1 - i)^2 + (1 - i)^3$  can be represented in the form  $a + bi$  where  $a$  and  $b$  are real values. Find  $|a - b|$ .
- A) 1                              B) 3                              C) 5                              D) 7                              E) NOTA
- 11) Which of the following is NOT a sixth root of unity?
- A)  $\frac{1}{2} - \frac{\sqrt{3}}{2}i$                       B)  $\frac{\sqrt{3}}{2} + \frac{1}{2}i$                       C)  $-\frac{1}{2} - \frac{\sqrt{3}}{2}i$                       D)  $-1$                               E) NOTA
- 12) How many of the following are NOT complex numbers?
- I.) 0  
II.)  $2i - 2$   
III.)  $8i$   
IV.)  $-\sqrt{2}$
- A) 1                              B) 2                              C) 3                              D) 4                              E) NOTA
- 13) Which of the following is NOT a fifth root of  $z = 16\sqrt{2} + 16i\sqrt{2}$  ?
- A)  $2\text{cis}(9^\circ)$                       B)  $2\text{cis}(81^\circ)$                       C)  $2\text{cis}(189^\circ)$                       D)  $2\text{cis}(297^\circ)$                       E) NOTA
- 14) Find the value of  $f(2 + i)$  if  $f(z) = iz^3 + 2z^2 - 3\bar{z} + 2i$ .
- A)  $-11 + 15i$                       B)  $-11 + 9i$                       C)  $2 + 18i$                       D)  $2 + 24i$                       E) NOTA
- 15) Let the complex number  $z_1 = a + bi$  where  $a$  and  $b$  are positive real numbers. Let the complex number  $z_2 = z_1^2$ . Which of the following statement(s) MUST be true?
- I.)  $|z_2| = |z_1|^2$   
II.) The angle that  $z_1$  makes with the positive real axis is half the angle at which  $z_2$  makes with the positive real axis.  
III.) If  $z_2 = c + di$ , then both  $c$  and  $d$  are positive.  
IV.)  $\frac{1}{z_2} = \frac{\bar{z}_1}{|z_1|^2}$
- A) I. only                      B) I. and II. only                      C) I. and IV. only                      D) All are true                      E) NOTA



- 16) Simplify:  $\frac{1}{i} + \frac{2}{i^2} + \frac{3}{i^3} + \frac{4}{i^4}$
- A) 4                      B)  $1 + i$                       C)  $2 + 2i$                       D)  $4 + 6i$                       E) NOTA
- 17) What is the norm of the complex number  $-6 - 8i$  ?
- A)  $-100$                       B)  $-10$                       C)  $10$                       D)  $100$                       E) NOTA
- 18) Evaluate:  $i^6 + i^{29} + i^{2006}$
- A)  $-3i$                       B)  $-i$                       C)  $1 + 2i$                       D)  $-2 + i$                       E) NOTA
- 19) Evaluate:  $\prod_{k=0}^{2N} i^k$  where  $N$  is an integer. Leave your answer in terms of  $N$ .
- A)  $i^{-N}$                       B)  $i^{-N/2}$                       C)  $i^N$                       D)  $i^{2N}$                       E) NOTA
- 20) Evaluate the sum:  $\sum_{k=0}^{2006} (-i)^k$
- A) 1                      B)  $i$                       C)  $-i$                       D)  $1 - i$                       E) NOTA
- 21) What is the real part of the following product?  $(5 - 6i)(2i + 1)$
- A)  $-7$                       B)  $-3$                       C)  $4$                       D)  $17$                       E) NOTA
- 22) Which of the value(s) for  $x$  makes the determinant equal to zero?  $\begin{vmatrix} x & -1 & 2 \\ 0 & i & 2 \\ -x & i & 2i \end{vmatrix}$
- I.)  $-1$                       II.)  $0$                       III.)  $i$                       IV.)  $-i$                       V.)  $1$
- A) II. only                      B) I., II., and V. only                      C) III. and IV. only                      D) All of these                      E) NOTA
- 23) Dave rips a piece of paper into four identical squares. He then takes the four numbers:  $5i$ ,  $-2$ ,  $300$ , and  $-2006 + 2007i$ , and writes one on each square paper. He puts all four squares of paper into a hat, shuffles them around, and randomly draws 2 squares without replacement. What is the probability that the product of the two numbers he draws is a real number?
- A)  $\frac{1}{12}$                       B)  $\frac{1}{6}$                       C)  $\frac{1}{3}$                       D)  $\frac{1}{2}$                       E) NOTA
- 24) What is the sum of the complex zeros to the quadratic function  $y = x^2 - 2x + 9$  ?
- A)  $-9$                       B)  $-2$                       C)  $2$                       D)  $9$                       E) NOTA



25) A multiplication by  $-i$  in the complex plane does what to a complex number,  $z = x + iy$  ?

- A) It rotates it  $90^\circ$  clockwise
- B) It rotates it  $90^\circ$  counter-clockwise
- C) It flips it across the  $y$ -axis
- D) It flips it across the  $x$ -axis
- E) NOTA

26) Evaluate the sum:  $\sum_{n=0}^5 n \cdot i^n$

- A)  $2 + 3i$
- B)  $3 - 2i$
- C)  $-2 - 3i$
- D)  $3 + 2i$
- E) NOTA

27) How many of the following statements are true regarding the complex function  $f(z) = |z|$  ?

- I.)  $f(-z) = f(z)$
- II.)  $f(\bar{z}) = f(z)$
- III.)  $f(z)$  is always real
- IV.) If  $f(z) = 0$ , then  $z$  can only be equal to 0.
- V.)  $f(z^2) = [f(z)]^2$

- A) 1
- B) 2
- C) 3
- D) 4
- E) NOTA

28) Two of the roots of a fifth order polynomial with integer coefficients,  $P(x)$ , are  $z_1 = 5 - 6i$  and  $z_2 = 2 + 3i$ . Which of the following statement(s) MUST be true regarding  $P(x)$ ?

- I.)  $P(x)$  has only 1 real root
- II.)  $z = -5 - 6i$  is a root of  $P(x)$
- III.)  $z = 2 - 3i$  is a root of  $P(x)$
- IV.) The product of all five roots of  $P(x)$  is a real number.

- A) IV. only
- B) I. and II. only
- C) I. and III. only
- D) I., III., and IV. only
- E) NOTA

29) Simplify:  $\sqrt{-2} \times \sqrt{-8} \times \sqrt{-16}$

- A)  $-16i$
- B)  $-8i$
- C)  $8i$
- D)  $16i$
- E) NOTA

30) What is the solution to the system of linear equations shown below?

$$\begin{aligned}x - iy + 2iz &= -1 \\ix + 2y + (2 - i)z &= 3i \\x + z &= 1\end{aligned}$$

- A)  $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ 4i \\ 0 \end{bmatrix}$
- B)  $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 - i \\ 1 \end{bmatrix}$
- C)  $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -11 + i \\ 1 + 8i \\ -i \end{bmatrix}$
- D)  $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 - i \\ -1 \\ i \end{bmatrix}$
- E) NOTA