National MAΘ 2008 SOLUTION KEY Mu Integration

1) $\int_1^3 (2x^2 - x - 4)dx = (2x^3/3 - x^2/2 - 4x)|_1^3 = 10 - 4 - 2/3 = 16/3$ (C)

2) $\frac{d}{dx}\int_a^x f(t)dt =$ f(x) represents the Fundamental Theorem of Calculus (E)

3) $\int_1^2 x6^{x^2-2}dx$ = (1/2) $\int 6^u du$ (U-Sub) = $(6^{x^2-2})/2\ln 6|_1^2 = 215/(2\ln 6)$ (A)

4) v(t) = $\int a(t)dt = \int e^{t/2}dt = 2e^{t/2} + C_1$; v(0) = 6 $\Rightarrow C_1 = 4$; s(t) = $\int (2e^{t/2} + 4)dt =$ $4e^{t/2} + 4t + C_2$; s(0) = 0 = 4 + C_2 $\Rightarrow C_2 = -4$, So s(4) - s(0) = $4e^2 + 12$ (D)

5) V= $\pi\int_0^{\pi} (e^{-x})^2 dx = \frac{-\pi}{2}\left(e^{-2\pi} - 1\right)$ (C)

6) $\int_1^4 |2 - x|\,dx = \int_1^2 (2-x)dx - \int_2^4 (2-x)dx =$ 2 - 3/2 - (0 - 2) = 5/2 (A)

7) $\int \frac{x}{(x-b)^2}dx = \int \frac{A}{(x-b)}dx + \int \frac{B}{(x-b)^2}dx \Rightarrow$ (Partial fractions) $\Rightarrow$ (A = 1, B = b)

$\ln|x-b| - \frac{b}{x-b} + K$ (D)

8) Since $3-y^2 = y + 1$ $\Rightarrow$ y = 1 or y = 2 $\Rightarrow$ A = $\int_{-2}^{-1}[(3-y^2)-(y+1)]dy = 9/2$ (A)

9) Given $\lim_{n\to\infty}\sum_{i=1}^{n} 3\left(-1-\frac{3i}{n}\right)^2\left(\frac{2}{n}\right)$, choose $x_i = i/n$, so a = x_0 = 0, b = x_n = 1, Δx = 1/n, and f(x) = $3(1 + 6x + 9x^2)$, thus the definite integral would be $6\int_0^1 (1 + 6x + 9x^2)dx$ (B)

10) $\int \sin^n x\,dx =$ (via Integration by Parts) $-\frac{1}{n}\cos x \sin^{n-1} x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$ (A)

11) y_{ave} = $\frac{1}{3}\left[\int_{-2}^{0}(2-x)dx + \int_0^1 (2+x)dx\right] = \frac{1}{3}\left[6 + \frac{5}{2}\right] = \frac{17}{6}$ (C)

12) $\int_0^1 \int_2^{\sqrt{y}} 2xy\,dx\,dy = \int_0^1 (xy^2)\Big|_2^{\sqrt{y}} dy = \int_0^1 (y^2 - 4y)dy$ = 1/3 -2 = -5/3 (D)

13) $\int \frac{2-x}{x^2+16}dx = 2\int \frac{1}{x^2+16}dx - \int \frac{x}{x^2+16}dx =$ (1/2)[$\tan^{-1}$(x/4) - $\ln(x^2+16)$] + C (C)

14) W = (force)(distance) = $\int_0^{10} \delta\pi(4)^2(10-y)dy =$ 800π(100 - 50) = 40,000π ft-lbs (A)

15) SA = $2\pi\int_a^b f(x)\sqrt{(1+(y')^2)}dx \Rightarrow$ SA = $2\pi\int_1^2 \ln(x^2)\sqrt{1+4/x^2}\,dx$ (A)

16) $\int_0^{\ln 10}\int_{e^x}^{10} \frac{1}{\ln y}dy\,dx = \int_1^{10}\int_0^{\ln y}\frac{1}{\ln y}dx\,dy$ (B)

17) $\int \arctan x\,dx$ = x arctanx - $\int \frac{x}{1+x^2}dx$ (Parts) ⇒ [x arctanx - .5 ln(1 + x^2)]$|_0^1$= .5(π/2-ln2) (D)

18) $M_x = m\bar{y} \Rightarrow$ M_x = m $\left(.5\rho\int_0((\sqrt{x})^2-(x^3)^2)dx\right)$/m = $\left(.5\rho\int_0((\sqrt{x})^2-(x^3)^2)dx\right)$ =5ρ/28 (B)

19) $\int \cos^2 t\,dt$ = $\int \frac{1}{2}(1+\cos 2t)dt) = \frac{1}{2}\left(t+\frac{1}{2}\sin 2t\right)+C$ (C)

20) f(x) = $\frac{d}{dx}(\ln(|\sec x(\sec x+\tan x|)+C) = \left|\frac{\sec x(\sec x\tan x+\sec^2 x+(\sec x+\tan x)(\sec x\tan x)}{\sec x(\sec x+\tan x)}\right|$

= (sinx + 1)/cosx (C)

21) $\int_{-2}^{4}$(2 [x] - 3|x|)dx = 6 + 4 + 2 - 2 - 4 - (6 + 24) = -24 (B)

22) L = $\int_a^b\sqrt{1+(y')^2}\,dx$ ⇒ L = $\int_0^1\sqrt{1+(\sinh x)^2}\,dx$ = $\int_0^1\cosh x\,dx$ = sinh1 - sinh0 = (e - e^{-1})/2 (D)

23) F(x) = $x^2\int_{x^2}^{0}\cos^2 t\,dt$ = -$x^2\int_0^{x^2}\cos^2 t\,dt$ & F'(x) = -x^2($\cos^2 x^2$)2x -2x $\int_0^{x^2}\cos^2 t\,dt$

= -2$x^3\cos^2 x^2$ -2x[x^2/2 + sin 2x^2/4] = -2$x^3\cos^2 x^2$ - x^3 - (xsin 2x^2)/2 (A)

24) $\int_2^{\sqrt{8}} x^3\sqrt{x^2-4}\,dx$ (U-Sub) = $\int_0^2 u^2(u^2+4)du$ = $\int_0^2(u^4+4u^2)du$ = 32/5 + 32/3 = 256/15 (C)

25) A = (1/2) $\int_0^{\pi/2}[(4\sin(2\theta)]^2\,d\theta$ = 8$\int_0^{\pi/2}\frac{(1-\cos 4\theta)}{2}d\theta$ = 2π - 0 = 2π (C)

26) $\int_3^4\frac{\ln(x-1)}{(x-1)}dx$ = (U-Sub & Parts) $\int_2^3 u^{-1}(\ln u)du$ = .5(ln u)^2 $|_2^3$ = $\frac{1}{2}\left[(\ln 3)^2-(\ln 2)^2\right]$ (B)

27) A = $\int_1^6 x^{-2}dx$ = 5/6, so 5/12 = $\int_1^c x^{-2}dx$ = -1/c + 1 ⇒ c = 12/7 (D)

28) $\int_0^4\frac{1}{x-1}dx$ (Improper Integral) = $\int_0^1\frac{1}{x-1}dx+\int_1^4\frac{1}{x-1}dx$; Since $\int_0^1\frac{1}{x-1}dx$ can be shown to diverge (-∞), it then follows that $\int_0^4\frac{1}{x-1}dx$ must also diverge. (E)

29) Γ(n) = (n - 1)! = $\int_0^\infty x^{n-1}e^{-x}dx$ (A)

30) $\int\frac{2x+1}{(x+5)^{100}}dx$ = (U-Sub) $\int\frac{2u-9}{u^{100}}du$ = $\frac{-1}{49u^{98}}+\frac{1}{11u^{99}}+C$ = $\frac{-1}{49(x+5)^{98}}+\frac{1}{11(x+5)^{99}}+C$ (C)