

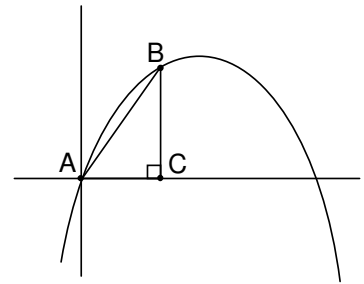
For all questions, choice E) NOTA denotes **None Of The Above** are correct.

- 1) Evaluate: $\lim_{x \rightarrow \infty} \frac{9x^3 - 3x^2 + 7x - 2007}{3x^3 + 6x + 2007}$
 A) -1 B) 0 C) 1 D) 3 E) NOTA

- 2) Given: $f(x) = 9x^2$ Evaluate: $\lim_{t \rightarrow 0} \frac{f(t+1) - f(1)}{t}$
 A) 0 B) 9 C) 18 D) 27 E) NOTA

- 3) Find the second derivative of $y = \frac{e^{x^2}}{2x}$ with respect to x ; $x \neq 0$.
 A) $y'' = \frac{\exp(x^2)(2x^2 + 1)}{2x^3}$ B) $y'' = \frac{\exp(x^2)(2x^2 - 1)}{2x^3}$
 C) $y'' = \frac{\exp(x^2)(x^4 - 2x^2 + 1)}{x^3}$ D) $y'' = \frac{\exp(x^2)(2x^4 - x^2 + 1)}{x^3}$ E) NOTA

- 4) A right triangle is constructed in the Cartesian plane with vertices A, B, C . A is the origin $(0,0)$, B is located in the first quadrant on the graph $y = 2x - x^2$, and C is a point on the x -axis with the same x -coordinate as B . Angle ACB is a right angle. Find the area of the triangle that has maximum area.
 A) $\frac{16}{27}$ B) $\frac{4}{3}$ C) $\frac{32}{27}$ D) $\frac{8}{3}$
 E) NOTA



- 5) Let $h(x) = \sqrt{x + \sqrt{x + \sqrt{x + \dots}}}$, $h(x) \geq 0$ Find $h'(2)$
 A) $\frac{1}{4}$ B) $\frac{1}{3}$ C) $\frac{1}{2}$ D) $\frac{3}{4}$ E) NOTA

- 6) Let $h(x) = \begin{cases} 9 - x^2, & x < 1 \\ a & x = 1 \\ x + 7 & x > 1 \end{cases}$

Which of the following describes the choice for the real number a so that $\lim_{x \rightarrow 1} h(x)$ exists?

- A) a must be equal to 8 for the limit to exist.
 B) There is no choice for a that will make the limit exist.
 C) a must be a positive real number for the limit to exist.
 D) a can be any real number for the limit to exist.
 E) NOTA

- 7) Evaluate $\frac{dy}{dx}$ of the curve $4x^3 - x^2y^2 + 2y^2 = 0$ at the point $(-1,2)$.
- A) $-\frac{5}{2}$ B) 0 C) 5 D) 8 E) NOTA
- 8) Over the domain $(1,4)$ the function $y = x^3 - 2x^2 + x - 1$ has which of the following properties?
 I) Concave up II) Concave down III) Increasing IV) Decreasing
 A) I only B) IV only C) I & III only D) II & III only E) NOTA
- 9) Given that the limit $\lim_{n \rightarrow \infty} a^n n^a$ converges to a finite value for some value of a , how many of the following CANNOT be equal to a ?
- I) -2 II) -1 III) $-\frac{1}{2}$ IV) $\frac{1}{2}$ V) 1 VI) 2
 A) 1 B) 2 C) 3 D) 4 E) NOTA
- 10) Find the minimum distance from the curve $x = y^2 + 3$ to the point $(2,-5)$
- A) 2 B) $2\sqrt{5}$ C) $\sqrt{34}$ D) $2\sqrt{10}$ E) NOTA
- 11) The volume of a sphere is increasing at a constant rate of 4π cubic meters per minute. Find the rate of change of the surface area of the sphere, in square meters per minute, when the radius is 2 meters long.
- A) 2π B) 4π C) 8π D) 16π E) NOTA
- 12) Evaluate: $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^{2n-3}$.
- A) $e^{-2} - 3$ B) e^{-2} C) $e^2 - 3$ D) e^2 E) NOTA
- 13) Find the maximum value of the function $h(x) = x^{1/x}$ for $x > 0$.
- A) 1 B) $\sqrt{2}$ C) $\sqrt[3]{3}$ D) ∞ E) NOTA
- 14) Let n be a positive integer. In terms of n find the coefficient of $\frac{d^n}{dx^n} [x^{n!}]$.
- A) $\frac{(n)!}{n!((n-1)!)}$ B) $\frac{(n)!}{(n!-n-1)!}$ C) $\frac{(n)!}{(n!-n)!}$ D) $\frac{(n)!}{(n!)-(n-1)!}$
 E) NOTA

- 15) $\lim_{x \rightarrow 3} \left(\frac{x}{3} + \pi - 1 \right) = \pi$. The $\varepsilon - \delta$ definition of the limit requires that for each $\varepsilon > 0$, there must exist a $\delta > 0$ for the limit to exist. Which of the following is the largest possible choice for δ to satisfy the definition when $\varepsilon = 0.3$?

A) $\frac{0.3}{\pi}$ B) 0.1 C) 0.3 D) 0.9 E) NOTA

- 16) Find the slope of the polar graph $r = 1 - \cos(2\theta)$ when $\theta = \frac{\pi}{3}$.

A) $-3\sqrt{3}$ B) $\frac{\sqrt{3}}{5}$ C) $\frac{3\sqrt{3}}{5}$ D) $\sqrt{3}$ E) NOTA

- 17) Let $g(x)$ be a continuous and differentiable function defined for all real numbers. The table shows the value of $g(x)$ and its derivative $g'(x)$ for certain values of x .

x	-2	-1	0	1	2	3
$g(x)$	-6	-2	0	2	0	-2
$g'(x)$	2	1	0	0	1	2

Which of the following MUST be true regarding the function $g(x)$?

- I) $g(x)$ is strictly increasing.
 II) $g(x)$ has a local minimum at $x = 1$.
 III) $g(x)$ has a critical point at $x = 0$.
 IV) $g(x)$ is concave up on the interval $(1,3)$.
- A) I only B) III only C) II & III only D) I, III, & IV only E) NOTA
- 18) Evaluate: $\lim_{n \rightarrow \infty} \frac{1}{n} [e^{1/n} + e^{2/n} + e^{3/n} + \dots + e^{n/n}]$
- A) $e - 1$ B) e C) $e + 1$ D) Does not exist E) NOTA

- 19) Given a continuous and differentiable function, f , with $f(0) = 1$, $f'(0) = 0$, $f''(0) = -1$, and $g(x) = (1 + f(x))^2$. Calculate $g''(0)$.
- A) -4 B) -2 C) 0 D) 2 E) NOTA

- 20) Given $F(x) = \int_0^x \sin(t^2) dt$. Over which interval is $F(x)$ NOT concave down?

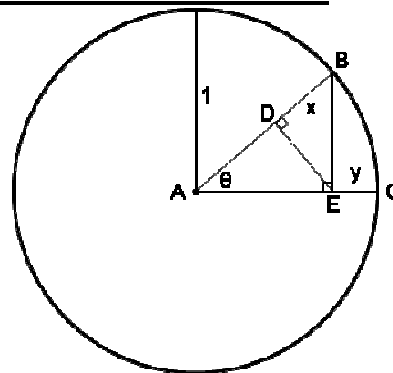
A) $\left(-\sqrt{\frac{\pi}{2}}, -\sqrt{\frac{\pi}{4}} \right)$ B) $\left(-\sqrt{\frac{\pi}{4}}, 0 \right)$ C) $\left(\sqrt{\frac{\pi}{4}}, \sqrt{\frac{\pi}{2}} \right)$ D) $\left(\sqrt{\frac{\pi}{2}}, \sqrt{\frac{3\pi}{4}} \right)$
 E) NOTA

- 21) In the figure the lengths of AB and AC are both equal to 1, the length of BD is denoted as x and the length of CE is denoted as y . BE is perpendicular to AC and DE is

perpendicular to AB . Evaluate: $\lim_{\theta \rightarrow 0} \frac{x}{y}$.

NOTE: Figure is not drawn to scale.

- A) 0 B) 1 C) 2
D) Does not exist E) NOTA



- 22) Evaluate: $\lim_{n \rightarrow \infty} \frac{1 + 2^n + 3^n + 4^n}{5^n}$
A) 1 B) 2 C) 10 D) Does not exist E) NOTA
- 23) Find the sum of the value(s) for c that satisfy the Mean Value Theorem for Derivatives of the function $f(x) = x^3 - x$ over the domain $[-2, 1]$.
A) -2 B) -1 C) 0 D) 1 E) NOTA
- 24) Evaluate: $\lim_{x \rightarrow -1} \frac{2x^2 - 3x + 1}{x + 1}$
A) -7 B) 0 C) 6 D) Does not exist E) NOTA
- 25) Using differentials approximate $\sqrt[3]{997}$ given that $10^3 = 1000$.
A) $\frac{997}{100}$ B) $\frac{499}{50}$ C) $\frac{999}{100}$ D) $\frac{2999}{300}$ E) NOTA
- 26) Let there exist a real function $f(x)$ such that $\lim_{x \rightarrow c} f(x)$ does not exist for some real number c , but $\lim_{x \rightarrow c} |f(x)| = L$ exists for some positive real number L . How many of the following MUST be true?
I) $\lim_{x \rightarrow c^+} f(x)$ and $\lim_{x \rightarrow c^-} f(x)$ both exist
II) $\lim_{x \rightarrow c} |f(x) - L| = 0$
III) $f(c)$ does not exist
IV) $\lim_{\Delta x \rightarrow 0} \frac{|f(c + \Delta x) - f(c)|}{\Delta x}$ exists
A) 1 B) 2 C) 3 D) 4 E) NOTA

- 27) If $\lim_{x \rightarrow \infty} \sqrt{Ax^2 + 12x - Bx} = 3$, find $A + B$, given $A, B > 0$.
A) 4 B) 6 C) 12 D) Does not exist E) NOTA
- 28) Given $f(x) = x^3 + x + 1$ and $g(x) = f^{-1}(x)$. Find $g'(3)$.
A) $\frac{1}{28}$ B) $\frac{1}{4}$ C) 4 D) 28 E) NOTA
- 29) Which of the following functions are differentiable at every value of x on the domain of all real numbers?
I) $y = x \cdot |x|$ II) $y = (x-1) \cdot |x|$ III) $y = |x-1|^3$ IV) $y = x^3 \cdot |x|$
A) II only B) I & IV only C) I, III & IV only D) All are differentiable
E) NOTA
- 30) Evaluate: $\lim_{x \rightarrow 0} \frac{x \sin(x)}{e^{x^2} - 1}$
A) -1 B) 0 C) $\frac{1}{2}$ D) 1 E) NOTA