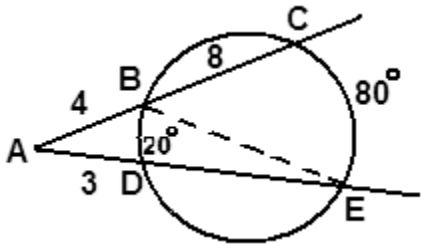
For all questions, answer E. NOTA means none of the above answers is correct. On this test, $i = \sqrt{-1}$.

1. In the figure below, arcs CE and BD measure $80°$ and $20°$, respectively. Segments AB, BC, AD measure 4, 8, and 3 cm respectively. Find the area of triangle ABE.



A. 12 B. 16
C. 32 D. 64 E. NOTA

2. One of the foci of the conic section with equation $-16x^2 + 9y^2 + 96x - 54y - 207 = 0$ is located at:

A. (3,-2) B. (3,3-$\sqrt{7}$)
C. (3+$\sqrt{7}$,3) D. (8,3) E. NOTA

3. Find the area of the quadrilateral formed by connecting the foci of the curves:

$$\frac{-(x-3)^2}{9} + \frac{(y-3)^2}{16} = 1 \text{ and } \frac{(x-3)^2}{25} + \frac{(y-3)^2}{16} = 1$$

A. $6\sqrt{7}$ B. 30
C. 45 D. $10\sqrt{41}$ E. NOTA

4. Find the volume of the ellipsoid formed after rotating $\frac{(x+4)^2}{9} + \frac{y^2}{4} = 1$ by $90°$ counter-clockwise and revolving it about $y = 0$.

A. 12π B. 16π
C. 18π D. 24π E. NOTA

5. The equation of the conic section determined by the polar equation $r = \frac{36\cos(\theta)}{9 - 5\sin^2(\theta)}$ can be written in the form $ax^2 + by^2 + cx + dy + e = 0$, where *a, b, c, d, e* are relatively prime integers and $a > 0$. Find $a + b + c + d + e$.

A. -23 B. 31
C. 41 D. 49 E. NOTA

6. Simplify: $\frac{cis(4\theta)}{cis(3\theta) * cis(6\pi + \theta)}$, where $0 \leq \theta < 2\pi$.

A. $\cos\theta + i\sin\theta$ B. $\cos\theta - i\sin\theta$

C. $\cos(\square)[2\theta) + i\sin(2\theta)$ D. $\cos(5\theta) + i\sin(5\theta)$ E. NOTA

7. If $\cot x = a$, $\cot y = b$ and $\csc x = c$, find $\cot\left(\frac{x}{2} - y\right)$ in terms of *a*, *b* and *c* for $abc \neq 0$ and $|a| \neq |b| \neq |c|$.

A. $\frac{bc - ab - 1}{b + c - a}$ B. $\frac{1 + ab - bc}{a - b - c}$

C. $\frac{b + c - a}{bc - ab - 1}$ D. $\frac{a - b - c}{1 + ab - bc}$ E. NOTA

8. After rotating the conic with equation $7x^2 + Axy + 13y^2 - 16 = 0$ by $30°$, the *xy* term is eliminated. Find *A*.

A. $-6\sqrt{3}$ B. $-2\sqrt{3}$

C. $2\sqrt{3}$ D. $6\sqrt{3}$ E. NOTA

9. Projecting vector $4i + 4j$ onto $2i + 5j$ yields the vector:

A. $3i + 3j$ B. $3i + 4j$

C. $\frac{7}{2}i + \frac{7}{2}j$ D. $4i + 5j$ E. NOTA

10. How many 'petals' does the graph of the curve $r = 8\cos^4\theta - 8\cos^2\theta + 1$ have?

A. 4 B. 8

C. 16 D. 24 E. NOTA

11. Given that $\sec 2\theta = \frac{x^2 + 1}{x^2 - x}$, find $\sin\theta$ for $0 \leq \theta \leq \pi$.

A. $\sqrt{\frac{2x^2 - x + 1}{2(x^2 + 1)}}$ B. $\sqrt{\frac{x - 1}{x^2 - 2}}$

C. $\sqrt{\frac{2x^2 - 2}{x + 2}}$ D. $\sqrt{\frac{x + 1}{2(x^2 + 1)}}$ E. NOTA

12. Find the eccentricity of the conic section determined by the polar equation $r = \pm 5\sqrt{\sec 2\theta}$

A. 0 B. 1
C. $\sqrt{2}$ D. $\sqrt{5}$ E. NOTA

13. The latera recti of $4x^2 + y^2 = 16$ pass through the coordinates:

A. $(0, \pm\sqrt{3})$ B. $(0, \pm 2)$
C. $(0, \pm 2\sqrt{3})$ D. $(0, \pm 2\sqrt{5})$ E. NOTA

14. The smallest angle θ between vector $\frac{\sqrt{2}}{2}i + \frac{\sqrt{2}}{2}j$ and unit vector *b* is $75°$. Find unit vector *b*, where *b* lies in the IV quadrant.

A. $-\frac{\sqrt{2}}{2}i + \frac{\sqrt{2}}{2}j$ B. $-\frac{1}{2}i + \frac{\sqrt{3}}{2}j$
C. $-\frac{1}{2}i - \frac{\sqrt{3}}{2}j$ D. $\frac{\sqrt{3}}{2}i - \frac{1}{2}j$ E. NOTA

15. When simplified, $\frac{(2+i)(3+i)(a+bi)}{(5+5i)} = 2 - i$. Find *a*+*b*, where *a* and *b* are integers.

A. -1 B. 0
C. 1 D. 2 E. NOTA

16. Find the axis of symmetry of the following curve: $x = y^2 + 4y + 5$

A. $x = 2$ B. $x = 17$
C. $y = -2$ D. $y = 17$ E. NOTA

17. Which of the following can be described as a dimpled limaçon?

A. $r = 5 + 2\sin\theta$ B. $r = 5 - 4\sin\theta$
C. $r = 5 - 5\cos\theta$ D. $r = 5 - 10\cos\theta$ E. NOTA

18. When solving for *x* that satisfy $\coth x = 4$, we get solutions of the form $x = a\ln b + k\pi i, (k \in \mathbf{R} \text{ and } a, b > 0)$. Find $a + b$.

A. $\frac{13}{6}$ B. 4
C. $\frac{9}{2}$ D. 8 E. NOTA

19. Which of the following could be the argument of $\sqrt{3 - i}$?

A. $-\frac{\pi}{3}$ B. $-\frac{\pi}{6}$

C. $\frac{\pi}{6}$ D. $\frac{\pi}{3}$ E. NOTA

20. Find the sum of the slopes of the asymptotes of the curve with equation $\frac{x^2}{36} - \frac{y^2}{16} = 1$

A. 3 B. $\frac{4}{3}$

C. 1 D. 0 E. NOTA

21. The range of $f(x) = \frac{\sqrt{2}}{2}\pi(\cos x - \sin x) - e$ can be written as $[a, b\ ]$, for $a, b \in \mathbb{R}$. Find *a*+*b*.

A. $\sqrt{2}\pi - 2e$ B. $\frac{\sqrt{2}}{2}\pi - e$

C. $\frac{\sqrt{2}}{2}\pi - 2e$ D. $-2e$ E. NOTA

22. Let $\cos(4\theta) = a\cos^4\theta + b\cos^2\theta + c$, where $a, b, c \in \mathbb{Z}$. Find $\arcsin(a + b + c)$.

A. $\frac{\pi}{2}$ B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$ D. $\frac{\pi}{4}$ E. NOTA

23. The convex polygon with vertices at $(4,1), (2,2), (3,4), (4,5), (x, y)$ has area $\frac{13}{2}$, where (x, y) passes through the line $y = 2x - 7$. Find $x + y$.

A. $\frac{-23}{2}$ B. 8

C. 11 D. 16 E. NOTA

24. Let $f(x) = \frac{\tan^2 x - \sec^2 x}{\tan^2 x + \sec^2 x + 2\tan x \sec x}$. Then $f(x)$ has:

I. Infinitely many vertical asymptotes
II. Exactly one point of discontinuity
III. Exactly one vertical asymptote

A. I B. II

C. I, II D. II, III E. NOTA

25. Which of the following is equivalent to $\text{Arctan}\frac{1}{2}+\text{Arctan}\frac{1}{3}$?

A. $\frac{\pi}{2}$ B. $\frac{\pi}{3}$

C. $\frac{\pi}{6}$ D. $\frac{\pi}{4}$ E. NOTA

26. Find the period of $f(x)=3\sin(6x)+4\cos(3x)$

A. $\frac{\pi}{5}$ B. $\frac{\pi}{6}$

C. $\frac{\pi}{9}$ D. $\frac{\pi}{18}$ E. NOTA

27. Find a coterminal angle for $\frac{512}{2008}\pi$:

A. $-\frac{689\pi}{251}$ B. $-\frac{438\pi}{251}$

C. $\frac{315\pi}{251}$ D. $\frac{817\pi}{251}$ E. NOTA

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28. A locus of points for which the sum of the distances to two fixed points is constant is called:

A. *circle* B. *parabola*

C. *ellipse* D. *hyperbola* E. NOTA

29. The name of the surface (in 3D) described by the equation $z=\frac{x^2}{a^2}-\frac{y^2}{b^2}$ is (where *x,y,z* are coordinates and *a,b* are constants):

A. *sphere* B. *paraboloid*

C. *ellipsoid* D. *hyperboloid* E. NOTA

30. Find *z* such that the vectors $\langle 1,2,2\rangle, \langle 3,4,z\rangle$ are perpendicular.

A. $-\frac{11}{2}$ B. -5

C. 5 D. $\frac{13}{2}$ E. NOTA