TAMPA BAY TECH INVITATIONAL CALCULUS TEAM MARCH 31, 2001

1. What is the x-coordinate of the point on the graph of $y = x^4 + x$ that is closest to the point (1,0)? (Round to the nearest hundredth)

2. Let $s(t) = 3t^4 - 52t^3 + 300t^2 - 672t + 15$, where $s(t)$ is the position function

   Let A = $a(3)$ where $a(t)$ is the acceleration function

   B = Average Velocity from $t = 1$ to $t = 8$

   C = Total distance traveled from $t = 1$ to $t = 8$

   Find $A + C - B$

3. Let $f(x) = 5x^4 - 3x^3 - 2x^2 + x - 1$

   Let

   A = $f'(2)$

   B = $f''(3)$

   C = $f'^{-1}(49)$ (for C, restrict the domain of $f(x)$ to positive integers)

   D = $f'''(3)$

   What is $AC + B - D$?

4. Find $\lim_{x\to\infty} \frac{2}{n}\left[e^{\frac{2n+2}{n}} + e^{\frac{2n+4}{n}} + e^{\frac{2n+6}{n}} + ... + e^{\frac{4n}{n}}\right]$ (exact answer please)

5. Let A = the first quadrant region bounded by the graphs of

   $y = 2x + 3$ and $y = 7$

   Let B = the volume when A is rotated about the line $x = -1$

   Let C = the volume when A is rotated about the line $y = 1$

   What is $\frac{C}{B}$?

6. What is $\underset{h\longrightarrow 0}{Lim} \frac{\ln(\ln(2+h)) - \ln(\ln 2)}{h}$?

7. Find the solution to $\frac{dw}{d\theta} = \theta w^2 \sin\theta^2$ when w(0)=1. Let A= $w\left(\frac{\sqrt{\pi}}{2}\right)$

8. A topless box is to be made from a piece of cardboard measuring 30 inches by 36 inches by cutting squares from the corners and folding the cardboard to form the box. What size squares should be cut from the corners of the cardboard to create a box of maximum volume?

9. Assume that Jeb is trying to make a circle and a square as cheaply as possible. He has a flexible 10 meter long cable that will be cut to form the circle and square. For reasons that defy explanation, each meter used in the perimeter of the square costs Jeb 2 dollars. The cable for the circle costs Jeb 2 dollars and 50 cents for each square. meter in the area of the circle. Assume he must use all 10 meters of the cable. If Jeb wants to pay as little as possible, what should the circumference of the circle be?

10. A, B, C, and D are the *approximate* values of the functions shown using a linearization centered at the given initial value.

$f(x) = x^2 + 3$ Initial: $x = 2$ A = f(2.1)

$g(x) = x^3 - 2x^2 + 3$ Initial: $x = 3$ B = g(2.9)

$h(x) = \dfrac{x^4 + x^3}{x^4}$ Initial: $x = 0.5$ C = h(.25)

$0 = xy + x^2 y + 18$ Initial: $x = 2$ D = y(0)

What is 10B - 10A – CD?

11. Let A= rate of change of the surface area of an ice cube when its volume is $8\ \text{cm}^3$ and it is melting at a rate of $4\ \text{cm}^3/\text{s}$

Let B= the rate at which the distance between a balloon and observer is changing when the balloon is rising vertically at a rate of 10 ft/s, the observer is 100 ft away from a spot directly below the balloon, and the balloon is 100 ft in the air?

Let C= the rate at which the top of a 15 ft ladder is moving on a vertical wall when the foot of the ladder is 5 ft away from the wall and is being pulled away at a rate of 2 ft/s?

What is ABC? (Omit all units)

12. A stone is dropped from the top of a 100 ft tall building. Two seconds later, a second stone is thrown downward from the top of the same building. With what velocity must it be thrown downward so that the two stones will strike the ground at the same time? (use g= -32 $\dfrac{ft}{s^2}$) Give your answer in ft/sec.

13. Find the area of the triangle bounded by the tangent to xy = 3 at (1,3) and the coordinate axis.

14. Find the area covered by the graph of r= $(2 + 2\cos\theta)$

15. A crane is lifting a 2750-lb car. The cable being used weighs 10 lb/ft. When the cable is attached to the car on the ground, 25 ft of cable hang from the crane. How much work is done in lifting the car 15 ft off the ground? (Please write units)