Algebra II Team Solutions March Regional

1. If h(x) = f(g(x)), then go backwards to find $h^{-1}(x) = g^{-1}(f^{-1}(x))$. Then $h^{-1}(5) = g^{-1}(f^{-1}(5)) = g^{-1}(2)$ [since f(2) = 5] , and $g^{-1}(2) = \boxed{7}$ [since g(7) = 2]

2. Write $f(x) = a(x-h)^2 + k$. Since b) is true, the parabola is symmetric about x=5, so h = 5. a) tells us $f(4)$ = 10 and c) tells us $f(0)$ = 16. Plugging in these values, we wind up with a system of equations in two variables (a and k). Solving, a = 1/4 and k = 39/4. Then $f(12) = 1/4(12-5)^2 + 39/4 = \boxed{22}$

3. x = 2^7 = 128 2y = 9^(1/2log$_9$y) = y^1/2→y = 1/4 z = 4/(1-1/2) = 8 128*1/4*8 = $\boxed{256}$

4. Summing all three, we have A+B+C = $\boxed{-15}$

5. f(5) = f(3(7/3)-2) = 1/(7/3) = 3/7 A = 3/7
5 = 1/(1/5) = f(3(1/5)-2) = f(-7/5) = f(k) , so B = -7/5
f(x) = f(3((x+2)/3) -2) = 1/((x+2)/3) = 3/(x+2) which is neither odd nor even
Thus, ABC = 3/7*-7/5*3 = $\boxed{-9/5}$

6. A) 4C2*(-4)^2 = 96
B) x^90 will be the term with the largest exponent, and it's easy to see there will be an x^89, an x^88, etc., through to the constant at the end, giving 91 terms
C) Letting x=y=1 will leave the sum of the coefficients in the expansion, so (3*1-1)^8 = 256
Thus, A+B+C = 96+91+256 = $\boxed{443}$

7. A) Minimum is where x^2 - 5 = 0, which does happen, so 9 - 4(0) = 9
B) Find the vertex, (2, -3). The max is then -3.
C) Let x^3 = u, then Y = u^2 - 2u + 5, and it follows the minimum of this is the minimum of the original. The vertex is at (1, 4), so minimum is 4. 9 - 3 + 4 = $\boxed{10}$

8. A) x^2+y^2 = (x + y)^2 – 2xy = 16^2 – 2*5 = 246
B) $3\sqrt{2}$
C) The circle given has radius 1, and since the ratio of the areas is equal to the ratio of the squares of the radii, it follows the ratio of the radii is the square root of the ratio of the areas, or $\frac{\sqrt{2}}{2}$ (since the ratio of the areas is ½). ABC = $246*3\sqrt{2}*\frac{\sqrt{2}}{2} = \boxed{738}$

9. We have 0 + 3x + 6y = 33 from the multiplication and 10y+9x–10x–27y = -17y-x = -71 from the determinant. Solving, $x=3$ and $y=4$, so xy = $\boxed{12}$.

10. A = 11C3 = 165 B = 8!/3!/2! = 3360 C = 9*9*8*7 = 4536, and A+B+C = 165+3360+4536 = $\boxed{8061}$

11. Because (10, 0) is on the ellipse, we know a = 10. Substituting the other point in our partial equation x^2/100 + y^2/b^2 = 1 yields b = 8. Then e = c/a = sqrt(a^2 - b^2)/a = 6/10 = 3/5. For part B, recall the definition of an ellipse…"the set of points the sum of

whose distances from two fixed points is a constant". This constant is 2a, and so the 2 sides of the triangle connecting the point on the ellipse to the foci will sum to 2a = 20. The third side will be the distance between the 2 foci, or 2c = 12. So perimeter = 32, and 3/5*32 = $\boxed{96/5}$

12. A) The intersections will occur where *f(x)* - *g(x)* = 0, and the x^4 term on each will be eliminated, leaving at most a cubic equation with at most 3 real roots, and so the maximum # of intersections is 3. (Students cannot argue ∞ because the functions are said to be distinct)
B) 6
C) Since the non-real roots occur in pairs, there are at most 8
D) Same reasoning as in part A). A cubic minus a quadratic leaves a cubic with at most 3 real roots, so maximum # is 3.
3*6*8*3 = $\boxed{432}$

13. We need only realize inverses are symmetric over y=x, so if the functions intersect, the intersection must be on that line, and it follows a=b, so a/b = $\boxed{1}$. (A quick check of the graphs on a TI-83 shows they do intersect at one place in the first quadrant.)

14. B/A = ln 2005! / log 2005! = ln 2005! / (ln2005!/ln 10) = ln 10
C = 1/(1 - (*e* - 1)/*e*) = 1/(1/*e*) = *e* C^(B/A) = *e*^ln 10 = $\boxed{10}$.

15. This is a standard problem where the region of possible x,y values is sketched and the vertices are tested to find the max. Our x and y values must satisfy 1) x ≤ 10, 2) y ≤ -x + 15, 3) y ≥ x - 6, and x,y positive. This gives us a polygonal region with vertices at (0,0), (0, 15), (6,0), (10, 4), and (10, 5). Plugging these values into Z, we see the max occurs at Z(10,5) = 50*10 + 30*5 = $\boxed{650}$.