Team Round -- March Algebra II Regional

1. Let $h(x) = f(g(x))$. Assume f, g, and h exist for all real values of x and each possesses an inverse function defined for all x. Using the values given below, find the value of $h^{-1}(5)$.

$f(5) = 11$	$g(2) = 3$
$f(4) = 2$	$g(7) = 2$
$f(3) = 7$	$g(11) = 4$
$f(2) = 5$	$g(5) = 1$
$f(1) = 8$	$g(8) = 5$

2. A quadratic function $f(x)$ has the following properties:
   a) When $f(x)$ is divided by $x - 4$, the remainder is 10
   b) $f(1) = f(9)$
   c) $f(0) = 16$

Find $f(12)$.

3. Find the value of xyz (with x, y, and z each greater than zero) if

$$16^{\frac{3}{4}} = x^{\frac{3}{7}}$$

$$y \log_3 9 = 3^{\log_9 y}$$

$$z = 4 + 2 + 1 + \frac{1}{2} + \ldots$$

4. Let all of the following be true:

$$A = 2B + 3C - 5$$
$$B = 3A - 3C - 3$$
$$C = -2B - 3A - 7$$

Find the value of A+B+C.

5. Let $f(3x-2) = 1/x$.

$A = f(5)$
B = the value of k where $f(k) = 5$
C = 5 if $f(x)$ is an odd function, 4 if even, 3 if neither

Find the value of ABC.

6. Let A = the coefficient on the x^2 term in the expansion of $(x-4)^4$

Let B = the number of terms in the expansion of $(x^2 + x + 1)^{45}$

Let C = the sum of the coefficients in the expansion of $(3x - y)^8$ (Hint: try substituting some particular values for x and y)

Find A+B+C .

7. Let A = the maximum value of $y = 9 - 4|x^2 - 5|$

Let B = the maximum value of $y = -x^2 + 4x - 7$

Let C = the minimum value of $y = x^6 - 2x^3 + 5$

Find A + B + C.

8. A = the value of $x^2 + y^2$ if $xy = 5$ and $x + y = 16$
B = the distance between the points (6, 3) and (3, 6)
C = the radius of a circle whose area is half that of the circle given by the equation $x^2 + y^2 + 2x - 8y + 16 = 0$
Find ABC.

9. Let $A = \begin{bmatrix} 5 & 3 & 1 \\ 9 & 2 & 2 \\ 0 & x & y \end{bmatrix}$. Let also A x $\begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 20 \\ 27 \\ 33 \end{bmatrix}$ and $|A| = -71$. Find the product *xy*.

10. Let A = the number of committees of three people able to be formed by choosing from a group of eleven people.
Let B = the number of distinct ways the letters in the word "AARDVARK" can be arranged.
Let C = the number of 4-digit codes able to be formed using the digits 0-9 if each digit can be used only once and the first digit cannot be 0.
Find A+B+C.

11. An ellipse centered at the origin (whose vertices lie on the coordinate axes) contains the points ($5\sqrt{3}$, 4) and (10, 0).
Let A = the eccentricity of this ellipse.
Let B = the perimeter of any triangle with both foci of the ellipse for two of its vertices and a third vertex anywhere on the ellipse (except the x-axis).
Find AB.

12. Let A = the maximum possible number of intersections of any two distinct quartic polynomials of the form $f(x) = x^4 + ax^3 + bx^2 + cx + d$.
Let B = the maximum number of times any sixth degree polynomial can cross the line $y = 15$.
Let C = the maximum number of non-real roots a polynomial of degree 9 with real coefficients can have.
Let D = the maximum number of times a cubic polynomial can intersect the parabola $y = x^2 + 3x + 1$.
Find ABCD.

13. Let $f(x) = x^3 - 2$ and let $g(x)$ be the inverse of $f(x)$. The graphs of the two functions $f(x)$ and $g(x)$ meet at the point (a, b). Find $\frac{a}{b}$.

14. Let A = log 1 + log 2 + log 3 + log 4 +…+ log 2005
Let B = ln 1 + ln 2 + ln 3 + ln 4 +…+ ln 2005
Let C = $1 + \frac{e-1}{e} + \frac{(e-1)^2}{e^2} + \frac{(e-1)^3}{e^3} + \dots$
Find $C^{B/A}$.

15. Find the largest possible value of $Z(X,Y) = 50X + 30Y$ if $(X+Y)$ must be less than or equal to 15, $(X - Y)$ must be no greater than 6, and X can be no greater than 10. ($X \geq 0$, $Y \geq 0$).