January Regional Calculus Exam

NOTA is defined as None of the Above Answers is Correct

1) Find a real number integer value of x that gives a vertical tangent line to the function $f(x) = e^{\sqrt[3]{x-1}}$.

A) -1 B) 0 C) 1 D) e E) NOTA

2) Using four rectangles on a regular partition of [0,2] calculate the **lower** sum approximation of $\int_0^2 (3x^2 + 2)dx$.

A) $\frac{37}{2}$ B) $\frac{37}{4}$ C) $\frac{61}{2}$ D) $\frac{61}{4}$ E) NOTA

3) Find the equation of the tangent line to $y = \arctan(2x)$ at $x = 0$.

A) $y = -2x$ B) $y = -4x$ C) $y = -4x + 2$ D) $y = 4x$ E) NOTA

4) If $f_0(x) = \left(\frac{x}{x+1}\right)$ and $f_{(n+1)} = f_0 \circ f_n$ for $n = (0,1,2...)$ Solve for $f_{49}(12)$. Put your answer in decimal form, **do not round,** and give the integer value of the 5^{th} decimal place.

A) 3 B) 6 C) 7 D) 8 E) NOTA

5) F(x) is defined to be $x^2 y'' - 4xy' + 6 = 0$. Which of the following equations are solutions to F(x)?

I. $y_1 = x^2$

II. $y_2 = x^2 + 1$

III. $y_3 = x^3$

IV. $y_4 = x^4$

A) I & II only
B) I & IV only
C) III only
D) I, III, & IV only
E) NOTA

6) The linear distance between Point A and Point B is 315 meters. At time t_1, A and B move directly towards each other until they meet at t_2. A third Point C starts in the same position as A and travels directly to B, then continues to go back and forth between A and B until A and B meet. Find the total distance traveled by C (in meters) on the interval of time $[t_1, t_2]$, given:
Magnitude of A's velocity: 35 m/s
Magnitude of B's velocity: 10 m/s
Magnitude of C's velocity: 1254 m/s

A) 315 B) 1254 C) 2205 D) 8778 E) NOTA

7) Given the derivatives $h' = 3\cos(x)\sqrt{\sin(x)}$ and $g' = \dfrac{1-x}{g^2}$ integrate to find the original equations of each using the initial conditions $h(\pi) = 4$, and $g(0) = 2$. Solve for h and g at $x = \dfrac{\sqrt{3}}{2}$. Your final answer is given by $W(x) = hg$, round to the nearest whole number.

A) 0 B) 11 C) 17 D) 35 E) NOTA

8) Given $y = C_1 e^{-x} + C_2 e^{x} + C_3 e^{2x}$ where C_1, C_2, C_3 are arbitrary constants. Using the following system of equations, find *y* (to the nearest whole number) given that $x = 1$:

$$C_1' e^{-x} + C_2' e^{x} + C_3' e^{2x} = 0$$
$$-C_1' e^{-x} + C_2' e^{x} + 2C_3' e^{2x} = 0$$
$$C_1' e^{-x} + C_2' e^{x} + 4C_3' e^{2x} = e^{5x}$$

A) 0 B) 1 C) 2 D) –72 E) NOTA

9) Let $W(y_1, y_2, y_3 \ldots y_n) = Det \begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y_1' & y_2' & \cdots & y_n' \\ y_1'' & y_2'' & \cdots & y_n'' \\ \vdots & \vdots & & \vdots \\ y_1^{(n-1)} & y_2^{(n-1)} & \cdots & y_n^{(n-1)} \end{vmatrix}$.

Given $y_1 = x$; $y_2 = x^2$; $y_3 = x^{-1}$. Find W to the nearest whole number when $x = 2$.

A) -3 B) -1 C) 3 D) 6 E) NOTA

10) Find the derivative of $y=[\tan^{-1}(\tan^{-1}(\tan^{-1}(x^2))]$ at $x=\dfrac{\pi}{\sqrt{2}}$. Give the thousandth digits as your answer.

A) 2 B) 3 C) 4 D) 5 E) NOTA

Questions 11 & 12 refer to Figure 1, below.

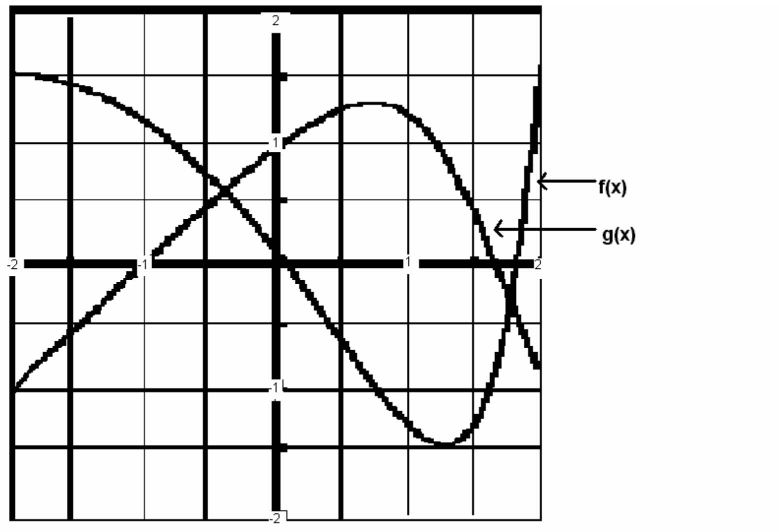

Figure 1

11) Define a function $h=\dfrac{\sqrt[3]{[f\circ g(x)]^2}}{-g'(x-1.7)}$ and estimate the value of $h\left(\dfrac{7}{10}\right)$. Round to the hundredth digit.

A) -.79 B) -.97 C) -1.31 D) 2.63 E)NOTA

12) True or False. For all true statements sum the integer values with in the parenthesis (next to the respective statement) to get the final answer. Please note: if the statement is false it is not included in your integer sum.

I. (3) The second derivative of $f(x)$ is negative on the interval of x values $\left(\dfrac{1}{2},\dfrac{13}{10}\right)$.

II. (7) $g\circ f(x)$ has a local maximum on the interval on the interval of x values $\left(-\dfrac{2}{10},-1\right)$.

III. (5) The slopes of $g(x)$ and $f(x)$ are equal at some value x in the interval of x values $\left(-\dfrac{1}{2},\dfrac{1}{2}\right)$.

A) 3 B) 7 C) 10 D) 15 E) NOTA

13) Use differentials to approximate $\tan\left(\frac{29\pi}{192}\right)$, given that $\tan\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}$. Give the fourth integer right of the decimal place.

A) 2 B) 5 C) 6 D) 9 E) NOTA

14) Solve: $\int_0^2 \left(\frac{4x^3+2x+1}{2x^4+2x^2+2x+1}\right)dx$. Give an exact answer.

A) $\frac{\ln(2)}{2}$ B) $\ln(2)$ C) $\frac{\ln(45)}{4}$ D) $4\ln(45)$ E) NOTA

15) Find the 112^{th} derivative of ln(x) at x=1 and divide this by 109!. Use only two significant figures in your answer.

A) -12000 B) $-13\text{x}10^6$ C) 73000 D) 12000 E) NOTA

16) Mikey has worked at a summer camp for 3 consecutive years. The first year he worked there were 100 total campers by the third year there were 300 total campers in the program. Assuming the number of campers is increasing exponentially, how many more years will Mikey have to work before there are 1000 campers?

A) 3 B) 4 C) 5 D) 7 E) NOTA

17) If $f(18)=5$ and $f'(18)=17$ use a linear approximation at $x=18$ to determine an approximation for $f(17.9)$. Give answer to the tenth place.

A) 2.0 B) 3.3 C) 5 D) 16.5 E) NOTA

18) The sum of two non negative real numbers x and y is equal to 108. What is the units digit of largest possible product of x^2 and y?

A) 2 B) 4 C) 7 D) 9 E) NOTA

19) Given $f(x)=\begin{cases} \dfrac{x^3+5x^2-38x-168}{x-6}, & x\neq 6 \\ k & x=6 \end{cases}$ What value of k makes f continuous on the interval $(-\infty,\infty)$?

A) -25 B) 6 C) 120 D) 130 E) NOTA

20) The radius of a circle is increasing at a rate of k, $k>0$.
Let r_1 = the radius of the circle where the rate of increase of the area of the circle is 2 times the rate of increase of the circumference.
Let r_2 = the radius of the circle where that rate of increase of the area of the circle is 5 times the rate of increase of the circumference.
Find $(r_1+r_2)r_1$.

A) 3.5 B) 7 C) 14 D) 160 E) NOTA

21) Consider $f(x)=\frac{1}{6}x^6+\frac{8}{5}x^5+3x^4-\frac{22}{3}x^3-\frac{29}{2}x^2+30x$ for values of x from (-3,3) find the sum of all the critical points on this interval.

A) -8 B) -3 C) -1 D) 0 E) NOTA

22) Solve $\lim_{x\to 0}[1-\sin(3x)]^{1/x}$. Give the hundredth digit of your answer. (Round your answer to the hundredth place).

A) 0 B) 1 C) 3 D) 5 E) NOTA

23) If $y=x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}$, find $\frac{dx}{dy}$ evaluated where $x=\sqrt{2}$. Round to the hundredths digit.

A) .08 B) .11 C) .22 D) .43 E) NOTA

24) Find the product of the magnitude of all possible tangents drawn from the point (1,-5) to the graph of $y=x^3+2x-7$. Round to the nearest whole number.

A) 5 B) 17 C) 20 D) 44 E) NOTA

25) Given $h(x) = f(g(x))$ and $g(1) = 3$; $g'(3) = 5$; $f(1) = 3$; $f'(3) = 13$. Find $\frac{h'(x)}{g'(x)}$ at $x = 1$.

A) 3 B) 5 C) 9 D) 13 E) NOTA

26) Find $\lim_{h \to 0} \frac{e^{2h+2} - e^2}{h}$ at the point $x = 1$. Round to the hundredth digit.

A) 2.72 B) 5.44 C) 7.39 D) 14.78 E) NOTA

27) Find the area under the curve $f(x) = \sin(x) + 2$ on the interval of x values $[0, \frac{\pi}{3}]$. Give answer to two decimal places.

A) 1.55 B) 2.09 C) 2.59 D) 6.78 E) NOTA

28) An Isosceles triangle is inscribed inside a parabola $y = x^2$ with one vertex of the triangle at the vertex of the parabola and the other two vertices lie on the parabola perpendicular to the parabola's axis. How fast is the area of the triangle changing when its height is 2 units if the height is increasing at a rate of 12 units per second?

A) $9\sqrt{2}$ B) 18 C) $18\sqrt{2}$ D) 36 E) NOTA

29) Solve for $y(2.2)$ given that $y = \int \frac{x}{e^x} dx$ and $y(2) = 4$. Give your answer to two decimal places.

A) .81 B) 4.05 C) 4.41 D) 4.81 E) NOTA

30) $\frac{dy}{dx} = \frac{1 - x^2}{y^2}$ Given $y(3)=0$ Find $y(1)$. Give two decimal places.

A) 1.39 B) 2.71 C) 2.88 D) 20 E) NOTA