January Calculus Regional Team Solutions:

Question #1: $V = x(45-2x)(33-2x) = 4x^3 - 156x^2 + 1485x$ then

$V' = 12x^2 - 312x + 1485 = 0$ The roots are $\dfrac{26 \pm \sqrt{181}}{2}$. Therefore the area is given as

$$\left(\frac{26-\sqrt{181}}{2}\right)^2 = \frac{\left(\sqrt{181}-26\right)^2}{4}.$$

Question #2: Let $n + a = b$ then $\lim_{x\to(b)^-}[x] = b-1$ and $\lim_{x\to(b)^-}|x| = b$, $\lim_{x\to(b)^+}[x] = b$ and

$\lim_{x\to(b)^+}|x| = b \Rightarrow$

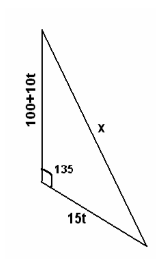
$$\left(\lim_{x\to(b)^-}(|x|-[x])\right) - \left(\lim_{x\to(b)^-}(|x|-[x])\right) + \left(\lim_{x\to(b)^+}(|x|-[x])\right) + \left(\lim_{x\to(b)}(|x|-[x])\right) =$$

$$(b-(b-1)) - (b-(b-1)) + (b-b) + (b-b) =$$

$$= 0$$

Question #3: $x^2 = (100+10t)^2 + (15t)^2 - 2(100+10t)(15t)(\cos 135) \Rightarrow$

$2x\dfrac{dx}{dt} = 20(100+10t) + (30)(15t) + \sqrt{2}(1500+300t) \Rightarrow t = 5$; $x = 209.844$; $\dfrac{dx}{dt} = 22.618$



Question #4: $\displaystyle\int_{-5}^{-1} -(x^3-x)dx + \int_{-1}^{0}(x^3-x)dx + \int_{0}^{1} -(x^3-x)dx + \int_{1}^{3}(x^3-x)dx = \frac{321}{2}$

Question #5: $V = \dfrac{4}{3}\pi r^3 \Rightarrow \dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt}$; $A = 4\pi r^2 \Rightarrow \dfrac{dA}{dt} = 8\pi r^2\dfrac{dr}{dt}$;

$$\frac{dV}{dt} = 10 = 4\pi\left(\frac{7}{5}\right)^2\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{125}{98\pi}; \ \frac{dA}{dt} = 8\pi\left(\frac{125}{98\pi}\right)\left(\frac{7}{5}\right) = \frac{100}{7}$$

Question #6: $\dfrac{1}{v-2} = \displaystyle\int_{5}^{v}(3x^2+16x-29)dx = \frac{1}{v-5}(v^3+8v^2+16v-29) \Rightarrow v^2+13v+36 = 157.$

$$v = \frac{\sqrt{653}-13}{2}.$$

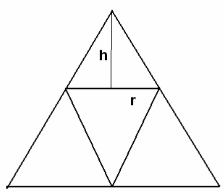
Question #7: $A = \int (\cos(x) - \sin(x))dx = \sqrt{2} - 1$; $B = \pi \int_0^{\pi/4} (\cos^2(x) - \sin^2(x))dx = \frac{\pi}{2}$;

$C = \sqrt{\sin\left[\cos\left(\sin\left(\frac{-\pi}{4}\right)\right)\right]}$ =.8301197; $D = 10.210176$ So the answer is given as .22

Question #8: This is a semicircle with radius 127 but we consider the radius to be 120 because that is the limit of integration so $\frac{1}{2}120^2\pi = 22619.47$

Question #9: Heights occur at x = 0, 5, 7.5, 10, 12.5, 15
Area is 2.5(.07069)= .17672 sum is 14

Question #10: $4x^3 = \frac{1}{16} \Rightarrow x = \frac{1}{4}$; $4 + y - 5 = 0 \Rightarrow y = 1$; $1 = \frac{1}{256} + k \Rightarrow k = \frac{255}{256}$



Question #11: Solution:
Let r= radius of the inner cone
Let 10-h equal the height from the base of the inner cone to the vertex of the outer cone

Similar triangles shows that $\frac{10}{4} = \frac{h}{r} \Rightarrow \frac{2h}{5} = r$

Volume of the inner cone = $\frac{1}{3}\pi\left(\frac{2h}{5}\right)^2(10-h)$ Differentiate and set equal to 0 with $h = \frac{20}{3}$ gives h of the inner cone = $\frac{20}{3}$ So with $\frac{10}{4} = \frac{h}{r}$ then $r = \frac{8}{3}$. The maximum volume is $\frac{1}{3}\pi\left(\frac{8}{3}\right)^2\left(\frac{20}{3}\right)$ or $\frac{640\pi}{81}$

Question #12: A: $f(x) = x^5 - 5x^4 - 143x^3 + 125x^2 + 4406x + 7280 = 0$ Sum of the zeros:
$13 + 7 - 2 - 5 - 8 = 5$

B: Intersection: -1.998357 $\int_{-1.998}^{0} x^2 dx = 2.661$

C: $\frac{d}{dx}\left(x^5 - 5x^4 - 143x^3 + 125x^2 + 4406x + 7280\right) = 5x^4 - 20x^3 - 429x^2 + 250x + 4406$ Set equal to zero and find values of *y* at each critical point… at *x*=3.4689 *f*(*x*) has a local maximum.

$A + B + C = 11.13$

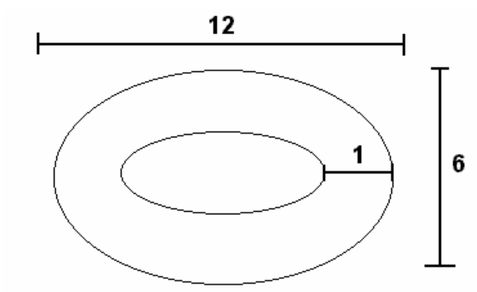
Question #13: The solutions of the parabola are 0 and 2h so the equation of the parabola is
$(x-0)(x-2h) = 0 \Rightarrow x^2 - 2hx = 0$

The coordinate of the turning point of the parabola is $\left\{\frac{2h}{2},\left[\frac{4(-1)0-(2h)^2}{4}\right]\right\} \Rightarrow \left\{h,\frac{-h}{2}\right\}$

Axis of sym. $x=h$ and $k=-h^2$ So the equation becomes $y=\frac{-k}{h^2}(x-h)^2+k$ then using the formula about tangent lines we get $\frac{y}{2}=\frac{-k}{h^2}(x-h)(-h)+k$ then substitute $x=h$ giving $y=2k$ and then the point of intersection is $(h,2k)$

Question #14: The two equations for the ellipses are: $1=\frac{x^2}{36}+\frac{y^2}{9}$ (outer ellipse) $1=\frac{x^2}{25}+\frac{y^2}{4}$ (inner ellipse). Solve both equations for *y*. $y_1=\left(9-\frac{9x^2}{36}\right)^{1/2}$; $y_2=\left(4-\frac{4x^2}{25}\right)^{1/2}$ Now

$$2\left[\int_{-6}^{6}(y_1)dx-\int_{-5}^{5}(y_2)dx\right]=8\pi$$



Question # 15:

Give the cookie as a circle $x^2+y^2=36$. Make a vertical cut extending through the first and fourth quadrants. Make the area of to the right of the cut in the first quadrant 6π. Solve $\int_{x}^{6}\sqrt{36-x^2}\,dx=6\pi$

Use a graphing calculator to find that $x=1.61$ Then $y=\pm 5.78$ so the cut goes from (1.61, -5.78) to (1.61, 5.78) so the total length is 11.56