January Calculus Regional **Team Questions**

1) Mr. Frazer wishes to make a topless box from a flat piece of cardboard with dimensions 45 ft by 33 ft by cutting squares from the corners and then folding the cardboard to form a box. Give the area of one of the removed squares in square feet, which maximizes the volume of his box? (Give an exact answer)

2) Let $[x]$ denote the greatest integer less than or equal to x. If n is a positive integer, then $\left(\lim_{x\to(n+a)^-}\left(|x|-[x]\right)\right)-\left(\lim_{x\to(n+a)^-}\left(|x|-[x]\right)\right)+\left(\lim_{x\to(n+a)^+}\left(|x|-[x]\right)\right)+\left(\lim_{x\to(n+a)}\left(|x|-[x]\right)\right)=?$
(Give an exact simplified answer in terms of a and n).

3) Daniel is 100 ft due North of Russell. Daniel is traveling North at 10 ft/s, and Russell is traveling SouthWest at 15 ft/s. How fast is the distance between Daniel and Russell changing 5 seconds later. (Give two decimal places in answer).

4) Evaluate $\int_{-5}^{3}|x^3 - x|dx$ (Give an exact answer).

5) How fast is the surface area of a sphere increasing when the radius is $\frac{7}{5}$ meters given that the volume is increasing at a rate of $10\frac{m^3}{s}$? (Give an exact answer in meters^2 per second)

6) The average value of $f(x) = 3x^2 + 16x - 29 = 157$ on the interval [5,v] where v>5. Find v. (Give an exact answer).

7) Let $f(x) = \cos(x)$ and $g(x) = \sin(x)$

Let A= the area bounded by the curves between $x = 0$ and $x = \frac{\pi}{4}$.

Let B= the volume of the solid obtained by rotating the region in A about the x axis.

Let C= $\sqrt{g \circ f \circ g\left(\frac{-\pi}{4}\right)}$

Let D= The value of x where the two functions intersect for the fourth time on the positive x axis.

Find $\frac{A+B}{C(C+D)}$. Give an approximate answer (rounded to the nearest hundredth digit).

8) Evaluate $\int_{-120}^{120}\sqrt{16129 - x^2}\,dx$. Round your answer to the hundredth place.

9) Using 6 rectangles on a regular partition of [0,15] calculate the lower sum approximation of $\int_{0}^{15}\frac{\ln(x+1)}{2^x}$ dx. Sum the first 3 digits to the right of the decimal place.

10) The line $16x + y - 5 = 0$ is normal to the curve $y = x^4 + k$. Give an exact answer.

11. One cone is inscribed within another one. The outer cone has height 10 and radius of 4. The inner cone is inscribed so that its apex lies on the base of the outer cone. The base of the inner cone is parallel to the base of the outer cone. The axes of the cones are collinear. Find the maximum volume of the inner cone. (Give an exact answer).

12) A function is given by $f(x) = x^5 - 5x^4 - 143x^3 + 125x^2 + 4406x + 7280$.
Let A= the sum of all the zeros of $f(x)$.
Let B= the area under the parabola $g = x^2$ bounded by the y-axis and the first point of intersection to the left of that axis. Give 3 decimal places.
Let C= The value of x which gives the largest local maximum value for $f(x)$. Save 3 decimal places.
Your answer is given as: $A + B + C$ (answer should have two decimal places)

13) A line and a parabola pass through the origin. The line is tangent to the parabola at the origin. The parabola reaches a maximum height of k and also passes through the point (2h,0). Find the intersection of the line and the axis of symmetry of the parabola in terms of h and k.

14) Given two ellipses the smaller one within the larger one with exactly overlapping minor and major axes (though the major axis and minor axis of the larger ellipse will extend past that of the minor one). Given that the larger ellipse has a minor axis of 6 ft and the major axis 12ft, and the major axis of the larger ellipse extends past the major axis of the minor ellipse by 1 ft on either side. Calculate the area of the major ellipse minus that of the smaller one. Give an exact answer in terms of feet^2.

15) Andy buys a cookie with a diameter of 12 inches. He wishes to cut the cookie so that his friend Nick will get twice as much of the cookie. He only makes one cut across the cookie… how long (in inches) is his single straight cut across the cookie (assuming that the cookie is a perfect circle with even thickness)? Round answer to hundredth place.