

1. $f(x) = 3^x = e^{x \ln 3}$ $f'(x) = \ln 3 e^{x \ln 3} = 3^x \ln 3$ **C**
2. Domain of $\sqrt{x^2 + 2x - 3} = \sqrt{(x+3)(x-1)} : (-\infty, -3] \cup [1, \infty)$
 Domain of $\ln(x-1) : (1, \infty)$ Common domain: $(1, \infty)$
 Range on domain: $(-\infty, \infty)$ **D**
3. $x = y^{2/3} = 8^{2/3} = 4$
 $2y \frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3x^2}{2y} \Rightarrow \left. \frac{dy}{dx} \right|_{(x,y)=(4,8)} = 3$ **B**
4. $\frac{1}{2-0} \int_0^2 3^x dx = \frac{1}{2} \left. \frac{3^x}{\ln 3} \right|_0^2 = \frac{8}{2 \ln 3} \approx 3.6$ **E**
5. $V = \int_1^3 2\pi(x+1)(x+1) dx = 2\pi \int_1^3 (x+1)^2 dx = 2\pi \left. \frac{(x+1)^3}{3} \right|_1^3 = \frac{112\pi}{3}$ **D**
6. $\frac{x^2 - 2x + 1}{\cos x}$ is continuous at $x = 0$, so $\lim_{x \rightarrow 0} \frac{x^2 - 2x + 1}{\cos x} = \left. \frac{x^2 - 2x + 1}{\cos x} \right|_{x=0} = 1$ **C**
7. $f'(x) = \frac{3}{5\sqrt{1-(x/5)^2}} \Rightarrow f'(4) = 1$ **D**
8. $\frac{dy}{dx} = 2^x \ln 2 + \cos x$ $y|_{x=3} = 8 + \sin 3$ $\left. \frac{dy}{dx} \right|_{x=3} = 8 \ln 2 + \cos 3$
 $y - y_1 = m(x - x_1) \Rightarrow y - (8 + \sin 3) = (8 \ln 2 + \cos 3)(x - 3)$
 $y(0) = (8 + \sin 3) - 3(8 \ln 2 + \cos 3) \approx -5.5$ **A**
9. *Perimeter* = 2(*height*) + *width* = $2h + w = 60 \Rightarrow w = 60 - 2h$
 $A = w \cdot h = (60 - 2h)h \Rightarrow \frac{dA}{dh} = 60 - 4h$
 Extrema: $\frac{dA}{dh} = 0 = 60 - 4h \Rightarrow h = 15 \Rightarrow A = 450$ **C**
10. $f'(x) = (2x)(\ln 3)3^{x^2+1}$ $f'(-1) = -18 \ln 3$ **E**
11. $f'(x) = -4x^3$ $f'(0) = 0$
 $f''(x) = -12x^2$ $f''(0) = 0$ Test is inconclusive for $f''(0) = 0$. **D**

12. Let $x = 2 \sin \theta \Rightarrow dx = 2 \cos \theta d\theta$ $x = 0 \Rightarrow \theta = 0$ $x = \sqrt{2} \Rightarrow \theta = \pi/4$

$$\int_0^{\sqrt{2}} \sqrt{4-x^2} dx = \int_0^{\pi/4} \sqrt{4-4\sin^2 \theta} 2 \cos \theta d\theta = 4 \int_0^{\pi/4} \sqrt{\cos^2 \theta} \cos \theta d\theta = 4 \int_0^{\pi/4} \cos^2 \theta d\theta$$

$$\dots = 2 \int_0^{\pi/4} [1 + \cos 2\theta] d\theta = 2 \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\pi/4} = \frac{\pi}{2} + 1 \quad \mathbf{B}$$

13. $\frac{f(\pi) - f(-\pi)}{\pi - (-\pi)} = \frac{\frac{1}{4+\pi} - \frac{1}{4-\pi}}{2\pi} = \frac{1}{\pi^2 - 16} \quad \mathbf{B}$

14. $3(1 - \log_{(\pi/2)}(\pi/2)) = 0$ $\cos(\pi/2) = 0 \Rightarrow \text{L'Hospital}$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{3(1 - \log_{(\pi/2)} x)}{\cos x} = 3 \lim_{x \rightarrow \frac{\pi}{2}} \frac{-1/x \ln(\pi/2)}{-\sin x} = \frac{3}{\sin(\pi/2) \cdot \pi/2 \cdot \ln(\pi/2)} \approx 4.23 \quad \mathbf{D}$$

15. $\text{Area} = \frac{1}{2} \int_{\alpha}^{\beta} [r(\theta)]^2 d\theta = \frac{1}{2} \int_0^{2\pi} [1 + \cos \theta]^2 d\theta = \frac{1}{2} \int_0^{2\pi} [1 + 2\cos \theta + \cos^2 \theta] d\theta =$

$$\dots = \frac{1}{2} \int_0^{2\pi} \left[1 + 2\cos \theta + \frac{1 + \cos 2\theta}{2} \right] d\theta = \frac{1}{2} \left[\theta + 2\sin \theta + \frac{\theta}{2} + \frac{\sin 2\theta}{4} \right]_0^{2\pi} = \frac{3\pi}{2} \quad \mathbf{B}$$

16. $I \approx \frac{1}{2} \frac{2}{4} [f(0) + 2f(.5) + 2f(1) + 2f(1.5) + f(2)]$

$$= \frac{1}{4} [0 + 2(.125) + 2(1) + 2(3.375) + 8] \approx 4.3 \quad \mathbf{C}$$

17. $(a, b) \Rightarrow b = 2a - 1 \Rightarrow a + b = 3a - 1$

$$y = 5 - x^3 \quad y(3) = -22$$

$$\frac{dy}{dx} = -3x^2 \quad \left. \frac{dy}{dx} \right|_{x=3} = -27 \quad m_{\perp} = -\frac{1}{dy/dx} = \frac{1}{27}$$

$$y + 22 = \frac{1}{27}(x - 3) \quad y = 2x - 1 \Rightarrow 2a - 1 = \frac{1}{27}(a - 3) - 22 \Rightarrow a \approx -10.7457$$

$$a + b = 3a - 1 = 3(-10.7457\dots) - 1 \approx -33.3 \quad \mathbf{A}$$

18. $f'(x) = 2 \sin(2x+5) \cos(2x+5)(2) = 4 \sin(2x+5) \cos(2x+5) \quad \mathbf{D}$

19. $\int (2x^3 + e^{2x}) dx = 2 \frac{x^4}{4} + \frac{e^{2x}}{2} + C = \frac{x^4 + e^{2x}}{2} + C \quad \mathbf{A}$

$$20. f'(x) = \frac{-2x}{2\sqrt{4-x^2}} \quad 4-x^2 > 0 \Rightarrow -2 < x < 2 \quad \mathbf{A}$$

$$21. \text{ Only highest order terms play a role approaching infinity: } \lim_{x \rightarrow \infty} \frac{8x^2}{2x^2} = 4 \quad \mathbf{B}$$

$$22. V = s^3 \quad \frac{dV}{ds} = 3s^2 \quad dV = 3s^2 ds \quad dV = 3(2)^2(.05) = .60 \quad \mathbf{B}$$

$$23. V = \int_1^3 (x^2)^2 dx = \int_1^3 x^4 dx = \frac{x^5}{5} \Big|_1^3 = \frac{242}{5} \quad \mathbf{D}$$

$$24. f'(x) = \frac{\ln x \cdot e^x - e^x \cdot 1/x}{(\ln x)^2} = \frac{e^x(x \ln x - 1)}{x(\ln x)^2} \quad \mathbf{C}$$

$$25. \frac{d}{dx} \int_a^{x^2} \sqrt{t} \cos t dt = 2x(\sqrt{x^2} \cos x^2) = 2x^2 \cos x^2 \Rightarrow [2x^2 \cos x^2]_{x=\sqrt{\pi}} = -2\pi \quad \mathbf{A}$$

$$26. \int (x + x^{-1}) dx = \frac{x^2}{2} + \ln |x| + C \quad \mathbf{A}$$

$$27. x = \int_0^2 (t^3 + \cos t) dt = \frac{t^4}{4} + \sin t \Big|_0^2 \approx 4.9 \quad \mathbf{E}$$

$$28. f(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases} \quad f'(x) = \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$$

$$\lim_{x \rightarrow 0^-} f'(x) = -1 \quad \mathbf{A}$$

$$29. x^2 + x + 6 \Big|_{x=2} = 12 \quad x - 2 \Big|_{x=2} = 0 \quad \frac{12}{0} \rightarrow \infty \text{ (does not exist)} \quad \mathbf{D}$$

$$30. f(x) = 2x^2 - 5x - 3 \quad f'(x) = 4x - 5 \quad f'(2) = 3 \quad \mathbf{A}$$