

## Question 1

- A. What is the greatest common factor of  $2x^2 - x - 91$  and  $4x^2 - 19x - 63$ ?
- B. What is the least common multiple of  $3x^2 - 23x + 14$  and  $x^2 - 4x - 21$ ?
- C. What is the conjugate of  $3x + 2$ ?
- D. What is the sum of all values of  $k$  for which  $4x^2 + kx + 1$  is a perfect square?

Please express  $\frac{B+D}{AC}$  in terms of  $x$ .

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## **February 2006**

## **Algebra I Team**

### Question 2

Evaluate the truth or falsity of the following statements. Add the numbers in parentheses next to each true statement. If the statement is false, do nothing – the value for that statement is zero. Give the resulting sum. (If there are no true statements, then the resulting sum is zero.)

If  $a$  is an integer, then  $-a^2$  must be negative. (-1)

All rational numbers have a reciprocal. (8)

Addition and multiplication are commutative, associative operations. (-27)

Multiplication distributes over subtraction. (64)

The set of positive integers is closed under division. (-125)

The product of a number and its additive inverse is always nonnegative. (216)

The square root of the square of a number always equals the number itself. (-343)

A number always equals itself. (512)

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A number always equals itself. (512)

## Question 3

- A. Consider the equation  $5.1d + b + c - 30ad = 0$ . Let  $a = 0.17$ . If  $b$  and  $c$  are nonzero constants and the equation is an identity, find the value of  $b$  in terms of  $c$ . What is the value of  $b/c$ ?
- B. How many values of  $x$  satisfy the following equation:  $\frac{62}{15}x + 1 = \frac{1}{5} + 4x + \frac{2}{15}x$ ?
- C. How many solutions does  $47(13x - 18) = 66 - 44x$  have?

What is  $(A + B)^C$ ?

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Question 4

- A. What is 65% of  $\frac{1}{2}$  of 360?
- B. What percent of 310 is 96.1?
- C.  $\frac{3}{10}$  is 20% of what number?
- D. Solve the following proportion:  $\frac{-42}{m} = \frac{12}{16-m}$ .

Please find  $A - B + C - D$ .

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## Question 5

Mark is taking a physics class for which he has two parts: a lecture and a lab. 25% of his overall grade comes from the lab; 75% comes from the lecture. In his lecture class, each of 3 tests is worth 20% of his grade, homework is worth 15%, and the final exam is 25%. In his lab course, 5 quizzes are collectively worth 20% of his lab grade and the rest of the lab grade will come equally from his 8 lab reports. Suppose in his lecture he has a homework average of 95 and he earned grades of 87, 98, and 94 on his tests. In his lab, he earned grades of 83, 96, 97, 95 and 99 on the quizzes and 86, 90, 91, 93, 95, 97, 100, and 100 on his lab reports. (All grades are out of 100.)

What is Mark's percentage in the lab component alone?

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What is Mark's percentage in the lab component alone?

## Question 6

Rachel and Jordan standing next to each other on one side of a circular lake and want to travel to the other side. Rachel swims directly across the lake with radius 1980 ft at 264 feet per minute. Jordan runs at 352 feet per minute on the 7040 feet trail around the lake and ends at the same destination point.

- A. How many minutes does it take Rachel to cross the lake?
- B. How many minutes does it take Jordan to run around the lake?
- C. At what speed, in feet per minute, should Jordan have run if he wanted to end at the same time as Rachel?
- D. At what speed, in feet per minute, should Rachel have swum if she wanted to cross the lake in 12 minutes?

Please find  $\frac{BD}{AC}$ .

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## Question 7

- A. What is the reciprocal of the slope of the line perpendicular to  $0.1y = \frac{11}{113}x + 7.45$ ?
- B. What is the slope of the line parallel to  $5.71x - 3.39y = 42$ ?
- C. What is the slope of the x-axis of a Cartesian Coordinate Plane?
- D. What is the value of  $k$  such that the line through  $(2, 3)$  and  $(4, k)$  has a slope of  $-k$ .

Please simplify  $\frac{C - AD}{B}$  into a fraction in lowest terms.

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## Question 8

Jeff is currently sixteen years old and 165 centimeters tall.

- A. If his bed must be at least 5 cm longer than he is tall, what is the set of all possible lengths in centimeters of his bed?
- B. If he wants to date women that are shorter than him by at least 2.5 cm but no more than 15 cm, what is the range of heights in centimeters of women he should date?
- C. If he grows 1.25 cm per year for the next 4 to 8 years inclusive and then stops, what is the range of his height in centimeters after he stops growing?
- D. When he grows old and frail, suppose he will shrink to within 5 cm of his height at age sixteen. What is the range of his decreased height in centimeters?

Please find  $((A \cap C) \cup B) \cup D$ .

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Please find  $((A \cap C) \cup B) \cup D$ .



## Question 9

Consider the following systems of equations.

$$\begin{array}{llll} y = \frac{4}{5}x + 7 & 4y = 4.5x - 148 & -50y = 79 + 12x & 17x + 41y = 3 \\ 2x + 2.8 = 2.5y & 1.125x - y = 37 & 25x = 6y - 78 & 205y = 85x + 15 \end{array}$$

- A. How many of the systems are consistent?
- B. How many of the systems are dependent?
- C. How many are inconsistent?
- D. How many are independent?

Please find  $AB + CD$ .

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## Question 10

- A. Please find the degree of the following monomial  $\frac{1}{18}z$  ?
- B. Please find the degree of the following monomial  $2xy$  ?
- C. Please find the degree of the following polynomial  $\pi m^5 + n^2$  .
- D. Please find the degree of the following polynomial  $7^2 a^2 b^2 + c^3$  .

Now, using the above answers consider the polynomial  $x^B - Dx^A - C$  .

Please factor it.

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## Question 11

- A. Find the discriminant of  $7x^2 - 31x + 341 = 0$ .
- B. How many distinct solutions exist for the equation  $x^2 - 23x + 102 = 0$ ?
- C. How many rational solutions exist for the equation  $323x^2 - 2x - 1 = 0$ ?
- D. What is the sum of the distinct solutions to the equation  $961x^2 + 186x + 9 = 0$ ?

Please find  $AD - BC$ .

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Please find  $\left(\frac{B}{A}\right)^C$ .

Question 12

Simplify:

$$\begin{pmatrix} \sqrt{50} & \sqrt{32} \\ 7\sqrt{3} & \sqrt{26} \end{pmatrix} + \sqrt{2} \begin{pmatrix} 3 & \sqrt{1} \\ \frac{12}{\sqrt{6}} & \sqrt{13} \end{pmatrix}$$

The answer will be in the form  $\begin{pmatrix} A\sqrt{B} & C\sqrt{D} \\ E\sqrt{F} & G\sqrt{H} \end{pmatrix}$ .

Please find  $A - B + C - D + E - F + G - H$ .

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## Question 13

The following are operations used for evaluating numerical expressions. Using the order of operations, rank them from first to last, numbering them from 1 to 4. Then, multiply the rank by the number in parenthesis next to the corresponding operation. Finally, please add all four products.

Addition/Subtraction (1)

Exponentiation (4)

Multiplication/Division (16)

Parentheses (25)

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Multiplication/Division (16)

Parentheses (25)

## Question 14

A. Find the result of the following expression  $\frac{18}{\frac{5}{\frac{2}{7}}} + 79(-3) - 7.2 \times \frac{1}{2}$

B. Find the additive inverse of the result of the following expression

$$\frac{-3}{4}[-6 + (-14)] - 217 \div (-31).$$

C. Find the multiplicative inverse (in fractional form) of the result of the following expression  $17.4(-6) + 101 - 11.4 \div 19$ .

D. Find the result of the following expression  $\left| -42 - 3(-11) - 20 \div \frac{1}{4} \right|$ .

Please find  $AC - BD$ .

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Please find  $AC - BD$ .

## Question 15

Consider the points  $J(4,1)$ ,  $K(9,3)$ ,  $L(13, -2)$  and  $M(3, -6)$ .

- A. Find the sum of the coefficients of the equation of the line written in standard form containing points  $J$  and  $L$ .
- B. Find the sum of the coefficients of the line written in standard form containing the points  $K$  and  $M$ .
- C. Find the sum of the coordinates of the point of intersection of line containing the points  $J$  and  $L$  and the line containing the points  $K$  and  $M$ .

Please find  $\left(\frac{B}{A}\right)^C$ .

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