1. $-\frac{8\sqrt{3}}{17}$ By the chain rule, the derivative of $f(x)=\arctan(\cos^2(x))$ is $\frac{1}{\cos^4(x)+1}\bullet(-2\sin(x)\cos(x))$. At $x=\frac{\pi}{3}$, the value is $\frac{16}{17}\left(-\sin(2\bullet\frac{\pi}{3})\right)=-\frac{8\sqrt{3}}{17}$

2. $\frac{1}{50}$ Due to properties of inverse functions, $g'(x)=\frac{1}{f'(g(x))}$. The value of x needed to compute the previous equation is found by $634=x^2+9$, which yields $x=25$. Therefore, $\frac{1}{2x}=\frac{1}{50}$.

3. $\frac{2}{\pi}$ Since the square is inscribed in the circle, the diameter of the circle is equal to the diagonal of the square. It then follows that $2r=s\sqrt{2}$ and $s=r\sqrt{2}$. The area of the circle, $A_C=\pi r^2$, changes at a rate of $dA_C=2\pi r dr$ while the area of the square, $A_S=2r^2$, changes at a rate of $dA_S=4rdr$. Thus, $\frac{dA_S}{dA_C}=\frac{4rdr}{2\pi rdr}=\frac{2}{\pi}$.

4. 2 By the product rule, the first derivative is $g'(x)=2xe^{-x}-x^2e^{-x}$. Likewise, the second derivative is $g''(x)=2e^{-x}-2xe^{-x}-2xe^{-x}+x^2e^{-x}$. Therefore, $g''(0)=2e^0=2$.

5. $\frac{\pi}{4}$ The graph of the function is the top half of a circle of radius 1 centered at the origin. Since the integral goes from 0 to 1, the area beneath the curve is one-fourth the area of the circle. $\frac{\pi}{4}$

6. 64 Cross multiplying so all like terms are on one side, we get $\frac{dy}{y}=-\frac{dx}{x}$. The result of integrating each side is $\ln y=-\ln x+C$. Our initial conditions show that $\ln 16=-\ln 16+C$, so $C=4\ln 4$. Finally, when x = 4, $\ln y=-\ln 4+4\ln 4=3\ln 4=\ln 64$. Therefore, y = 64.
7. 1 The first derivative of the function reveals critical points, so $f'(x)=x^3-2x^2-5x+6$, whose roots are 1, -2, and 3. The second derivative reveals whether these critical points are minimums, maximums, or neither, so $f''(x)=3x^2-4x-5$. The second derivative is positive when x = -2 and x = 3 (local minimum) but negative when x = 1 (local maximum). Therefore, 2 – 1 = 1.
8. 10 The Frisbee is 18 feet above the initial position after 3 seconds. The path of the Frisbee follows the hypotenuse of a right triangle. If "x" is the Frisbee's horizontal distance from Spencer, "y" is the vertical distance, and "F" is the absolute distance, then $x^2+y^2=F^2$ and $xdx+ydy=FdF$. When y =18, x = 24 and F = 30, $dF=\frac{18(6)+24(8)}{30}=10$.
9. 0 If δ is a constant, then the definite integral will also be a constant. The derivative of a constant is always 0.

10. 8 $\xi(x)$ is clearly the sum of an infinite geometric series with $a_1 = \sin x$ and $r = -\cos x$. The sum of such a series is $\frac{a_1}{1-r} = \frac{\sin x}{1+\cos x}$. Using a u-sub of $u = 1+\cos x, du = -\sin x dx$, our new integral becomes $\int_{3/2}^{1} -\frac{du}{u} = -\ln 1 + \ln\frac{3}{2} = \ln 3 - \ln 2$. Thus $2^3 = 8$.

11. $100e^9$ To solve the differential equation, arrange all like terms on one side like so: $\frac{dP}{P} = 2t dt$. Integrating both sides, we get $\ln P = t^2 + C$. Our initial conditions show that $\ln 100 = 0 + C$. Therefore, $\ln P = 9 + \ln 100$ so $P = 100e^9$.

12. $\frac{16\pi}{15}$ The problem is rotating the graph bounded by $y = 1 - x^2$ and the x-axis around the x-axis. Using the disk method, the desired integral is $\pi\int_{-1}^{1}(1-x^2)^2 dx$ which is equivalent to $\frac{16\pi}{15}$.

13. $\frac{19}{12}$ We need to simplify the infinite radical before integrating. We can see that $y = \sqrt{x+y}$ or $y^2 - y = x$. Completing the square, we get $\left(y - \frac{1}{2}\right)^2 = x + \frac{1}{4}$ and $y = \sqrt{x+\frac{1}{4}} + \frac{1}{2}$ (the square root must be positive because y must always be positive). Thus, our integral is $\int_0^2 \sqrt{x+\frac{1}{4}} + \frac{1}{2} dx = \frac{2}{3}\left(x+\frac{1}{4}\right)^{3/2} + \frac{x}{2}\Big|_0^2 = \frac{19}{6}$. Since the question asks for average value, we divide by two to get $\frac{19}{12}$.

14. 10 The point on the graph will be (A, A^2) and the slope of the line tangent to the graph is *2A*. Thus, the equation of the line is $y - A^2 = 2A(x-A)$ or $y = 2Ax - A^2$. The x-intercept is $\frac{A}{2}$ and the y-intercept is $-A^2$. The area of the triangle is $.5\left(\frac{A}{2}\right)\left(A^2\right) = \frac{A^3}{4} = 2$. Therefore, A = 2 and B = 4 and thus 3B – A = 10.

15. 160π Pappus's Theorem states that the volume of a revolved region is equal to $2\pi MN$, where M is the area of the region and N is the distance from the centroid of the region to the line of revolution. Simple geometry shows that $M = 10$. The centroid is the average of the x and y-coordinates of the vertices, so $(\bar{x}, \bar{y}) = (0, -4)$, whose distance to y = 4 is 8. Therefore, the volume is $2\pi(8)(10) = 160\pi$.

16. -3 By the mean value theorem, $\frac{g(3)-g(0)}{3-0} = g'(c) = 3c^2 - 12$. Since $\frac{g(3)-g(0)}{3-0} = -3$, the answer is -3.

17. $\frac{56}{3}$ Looking at the equation for y, we see that the integral is a constant. Therefore, the particle only travels in the x-direction. Therefore, the distance traveled in the x-direction is simply the difference between the beginning and end points: $\frac{64}{3}-\frac{8}{3}=\frac{56}{3}$

18. $-\frac{1}{e^2}$ Taking the tangent of both sides, we get $xy=1$. Implicit differentiation yields the derivative as $\frac{dy}{dx}=-\frac{y}{x}$. At the point $\left(e,\frac{1}{e}\right)$, $\frac{dy}{dx}=-\frac{1}{e^2}$.

19. $\sqrt{2}$ From basic properties of integrals, we know that if $f(x)$ is an odd function, then $\int_{-a}^{a} f(x)dx=0$. The function $f(x)=e^{x^2}\sin x$ is odd, so that leaves us with $\int_{-\pi/4}^{\pi/4}\cos x dx=\sqrt{2}$

20. $\frac{4\pi}{15}$ The intersections of the graphs occur at x = -1, x = 0, and x = 1. When rotated around the y-axis, the region between x = -1 and x = 0 will have the same volume as the region from x= 0 to x = 1 and will also overlap each other. Therefore, since using the shell method once will account for the total volume, we must evaluate the integral $2\pi\int_0^1 x(x-x^3)dx$, which is equivalent to $\frac{4\pi}{15}$.

21. $\frac{1}{9}$ The first step is to find the area of the dartboard, which is $\int_0^3 9x-3x^2dx=\frac{27}{2}$. The area desired lies between the graphs $y=9x$ and $y=3x^2$ between x = 0 and $x=\frac{1}{3}$ and between the graphs $y=3$ and $y=3x^2$ between $x=\frac{1}{3}$ and x = 1. Thus, this area is $\int_0^{1/3} 9x-3x^2dx+\int_{1/3}^{1} 3-3x^2dx=\frac{81}{54}$. Finally $\frac{\frac{81}{54}}{\frac{27}{2}}=\frac{1}{9}$

22. 1 At x = 0, the derivative of f(x) will equal zero for all powers of tan(x) except k = 1 (all other powers will have tan(0) in the derivative. Thus, $\sec^2(0)=1$.

23. 18 The limit is clearly the difference quotient for derivatives. Therefore, $f'(x)=3x+6$ so $f(x)=\frac{3x^2}{2}+6x+C$. The initial conditions show that C = 0, so $f(2)=\frac{3(2)^2}{2}+6(2)=18$.

24. 2 We want to find the value of x that minimizes S(x). Therefore, $S'(x)=\frac{(3x-5)(3e^{3x})-(e^{3x})(3)}{(3x-5)^2}=0$. The numerator is equal to zero when x = 2.

25. 4 Examination of the numerator reveals that it is equivalent to $\left(\tan x+x\right)^2$. Therefore, we have $\lim_{x\to 0}\frac{\left(\tan x+x\right)^2}{x^2}=\lim_{x\to 0}\left(\frac{\tan x+x}{x}\right)^2$ which is logically equivalent to $\left(\lim_{x\to 0}\frac{\tan x+x}{x}\right)^2$. Using L'Hopital's rule, our inside limit is $\lim_{x\to 0}\frac{\sec^2 x+1}{1}=2$. Thus, $2^2 = 4$.