2008 FAMAT State Convention Statistics Bowl Solutions

1. $\frac{\mathbf{-2 \pm \sqrt{19}}}{\mathbf{3}}$. The mode of the distribution is 3, the median is 4 and the mean is 5. So the quadratic equation is $3x^2 + 4x - 5 = 0$. Solving the equation by the quadratic formula gives $\frac{-4 \pm \sqrt{16 - 4(3)(-5)}}{2(3)} = \frac{-4 \pm \sqrt{76}}{6} = \frac{-4 \pm 2\sqrt{19}}{6} = \frac{-2 \pm \sqrt{19}}{3}$.

2. **(25.4951, 28.9049)**. In order to find the interval, we must perform a t confidence interval. The formula is $\bar{x} \pm t\frac{s}{\sqrt{n}}$. Using the t-table, with a degree of freedom of 24, the t value is 2.064. Plugging the other values in gives $27.2 \pm (2.064)\frac{4.13}{\sqrt{25}}$, which leads to 27.2 ± 1.704864 = 25.495136 to 28.904864. Rounding both values to four decimal places gives the final result.

3. $\frac{\mathbf{13}}{\mathbf{18}}$. 40% of students drive and 65% of the drivers are boys, so 26% of the population are boy drivers, which is part A. 35% of the drivers are girls, so 14% of the population are girl drivers, therefore 36% of the population are girls who don't drive, which is part B. Therefore, the value of the expression is $\frac{.26}{.36} = \frac{13}{18}$.

4. $\frac{\mathbf{20809}}{\mathbf{257}}$. In order to find the mean and standard deviation, we must set up two equations using the information given. Those equations are $\frac{65 - mean}{sd} = -.52$ and $\frac{92 - mean}{sd} = 2.05$. By cross multiplying both equations and then eliminating the mean gives a standard deviation equal to $\frac{2700}{257}$. Plugging this result in gives a value for the mean of $\frac{18109}{257}$. The sum of the two numbers gives the solution.

5. **2.0859**. The chart including totals is as follows:

		Both parents	One parent	Neither parent	total
Student	Yes	415	311	254	980
(Do you smoke?)	No	235	499	316	1050
total		650	810	570	2030

For Part A, the probability is $\frac{980}{2030} = \frac{14}{29}$. For part B, the probability is $\frac{650}{2030} = \frac{65}{203}$. For part C, the probability is $\frac{980}{2030} + \frac{810}{2030} - \frac{311}{2030} = \frac{1479}{2030} = \frac{51}{70}$. For part D, there are 570 students in which neither parent smokes. So the answer is $\frac{316}{570} = \frac{158}{285}$. The sum of the four fractions is exactly 2.085913059, which when rounded to four decimal places, gives you the solution.

6. **.9931**. First we must find the raw score by using the formula $z = \frac{raw - mean}{sd}$. Using the z value from the table and the values given produces $1.645 = \frac{raw - 500}{\frac{100}{\sqrt{750}}}$. Solving gives a raw score of 506.0066907. Plugging this in against the alternative mean produces $z = \frac{506.0066907 - 515}{\frac{100}{\sqrt{750}}} = -2.462919181$. Using the table value of -2.46 produces a proportion of .0069. Since the alternative hypothesis is greater than, the power is $1 - .0069 = .9931$.

7. $\mathbf{\frac{\sqrt{74}}{37}}$. To find the values of a and b in the distribution, add the frequencies and find the partial mean. There are 30 tests unaccounted for, so a +b =30. Score (Frequency) for 1, 2, and 3 produces a total of 100. Since the mean = 3, 125 is left over. So, 4a + 5b =125. Solving the two equations gives values of a = 25 and b = 5. Once you have the distribution complete, subtract the mean from each value, square the differences, multiply the squared differences by their frequencies, add them up. You get a total of 100. Divide by 74 and you get 50/37. The square root of the fraction is $\frac{5\sqrt{74}}{37}$, which is C. When you plug the numbers in, you get $\frac{5\left(\frac{5\sqrt{74}}{37}\right)}{25} = \frac{\sqrt{74}}{37}$.

8. $\mathbf{\frac{-507}{1600}}$ or **-.316875**. The line of best fit equation is $y - \bar{y} = r\left(\frac{s_y}{s_x}\right)(x - \bar{x})$. Plugging the numbers in gives $y - 3.2 = .675\left(\frac{.4}{12}\right)(x - 106)$. Solving for y gives $y = .0225x + .815$. So A=.0225 and B=.815. Part C is r squared, which is .455625. For part D, plug 102 in for IQ and get a predicted G.P.A. of 3.11. So the residual is 1.50-3.11 = -1.61. So the expression answer is .0225+.815+.455625 – 1.61= -.316875.

9. **-12.8**. First, you must change the standard deviation by multiplying by 1.6, which is A. When you multiply the original mean by 1.6, you get 100.8. You must subtract 25.8 to get to the new mean of 75, so B= -25.8. So the transformation equation is y=1.6x - 25.8. Plugging in John's score of 72 gives a new score of 1.6(72) -25.8 = 89.4. Plugging in Jane's new score of 99 gives an original score of 99=1.6x-25.8 = 78. Plugging into the expression gives 1.6 – 25.8 +89.4 – 78 = -12.8.

10. $\mathbf{\frac{36483}{2350}}$. The number of degrees of freedom is (n-1), or 4, in this problem. To find the $\chi^2 = \sum \frac{(obs - \exp)^2}{\exp}$. So the answer is $\frac{(105-94)^2 + (92-94)^2 + (81-94)^2 + (91-94)^2 + (101-94)^2}{94} = \frac{176}{47}$. The critical value

with a degree of freedom of 4 at the 10% level is 7.78. The sum of the three numbers is the result.

11. **29**. Plugging the values into a Venn diagram leaves three open spaces. Each space represents those students who like exactly two of the three foods. Let a= hamburgers and fish sticks, b=fish sticks and macaroni and cheese and c= macaroni and cheese and hamburgers. The equations from the totals of each food are: a +b =4, b +c =10, and a + c=8. Solving these equations produces a = 1, b = 3, and c = 7. So part A= 11. When you add up all the numbers in the venn diagram, the total is 30. For part C, the answer is 5+3+4 = 12. So, the final answer is 11 + 30 – 12 = 29.

12. **15.32**. Part A of the question is np, which is 40(.3)=12. Part B of the question is $\sqrt{np(1-p)}$, which in this case is $\sqrt{40(.3)(.7)} = 2.898275349 \approx 2.90$. Part C of the question is a binomial situation, which is ${}_{40}C_{10}(.3)^{10}(.7)^{30} = .1128173016 \approx .11$. Part D of the question is a cumulative binomial situation, which in this case is $binomcdf(40,.3,10) = .308742722 \approx .31$. 12 + 2.90 + .11 + .31 = 15.32.