1. C – $a + 1/a = b + 1/b \Rightarrow a - b = \frac{a-b}{ab} \Rightarrow ab = 1 \Rightarrow a^2 + b^2 + 2ab = 10 + 2 = 12$

2. E – No arithmetic is needed. These circles are centered on the line $y = 0$ and their two intersections are thus centered about the line $y = 0$ as well. So $y_1 = -y_2,\ y_1 + y_2 = 0$.

3. C – Then the side length must be $j+1$ if $j+1>0$ or $-j-1$ if $j+1<0$, thus it is necessarily $|j+1|$.

4. D

5. D – Revenue is $xP(x) = 400x - 10x^2$ is maximized at $x = 20$

6. B – This can be thought of as the top half of the ellipse given by $x^2/4 + y^2/16 = 1$, which has semi-axes of lengths $2,4$. So we take $(1/2)\pi ab = 4\pi$

7. C – Use synthetic division to find the roots: $(x-3)(x-2)(x^2+4x+6) = 0$; the quadratic factor has complex roots

8. D – $(1/2)ab\sin C = (1/2)(1)(1)\sin(120^\circ) = \sqrt{3}/4$

9. B – So 10 Kat's work at 1 cookie per hour. Thus 10 Kat's will eat 5 cookies per 5 hours.

10. C –$1+2+3+4+6 = 16 > 12$

11. E – $2x + 2y + 5z = 6 = 2(x+y+2z) + z = 2(3) + z = 6 + z$, so $z = 0$, so $2x + 2y = 6,\ \ x + y + z = 3$

12. C – $a = kb/c^2,\ 2 = 3k/9,\ k = 6,\ a = 6(7)/9 = 14/3$

13. B – $9(1) + 90(2) + 109(3) = 516$

14. A – For the centroid, take the average of the coordinates: $(1/3)[(3,-2) + (2,-3) + (1,8)] = (2,1)$

15. A – $\cos a = \frac{\langle 6,2,3\rangle \cdot \langle 4,3,-12\rangle}{|\langle 6,2,3\rangle| \cdot |\langle 4,3,-12\rangle|} = \frac{-6}{(\sqrt{169})(\sqrt{49})} = -6/91$

16. D – Add each equation to get $2(x_1 + x_2 + ... + x_n) = (1/2)(n)(n+1)$, so $x_1 + x_2 + ... + x_n = (n^2 + n)/4$. Also, we can add all the odd-numbered equations: $x_1 + x_2 + ... + x_n = 1 + 3 + ... + (n-1) = (n/2)(n/2)$. Therefore, since $(n^2+n)/4$ must equal $(n/2)(n/2)$, we have $n = 0$ but it's not both positive and even.

17. D – $P(Lost\ third\ |\ Won\ 2\ or\ 3) = \frac{P(WWL)}{P(WWL) + P(WLW) + P(LWW) + P(WWW)} =$

$$\frac{(1/2)(1/4)(7/8)}{(7/64) + (3/16) + (1/16) + (1/64)} = 7/24$$

18. E – This simplifies to $\sin x = 0,$ or $\cos x = 0$, which has 4 solutions: $x = 0, \pi/2, \pi, 3\pi/2$

19. D – The difference between polynomials is a polynomial of the higher degree. Therefore, we have a polynomial of degree 21, it can have 21 roots, 20 roots, 19 roots, etc. So $1 + 2 + 3 + ... + 21 = 231$

20. B – This forms a cone with radius 3 and height 2. So $V = (1/3)(3^2)(2)\pi$

21. A – $\cos(x/2) = \sqrt{\frac{1+\cos x}{2}} = \sqrt{.64}$

22. A – $1/1^2 + 1/3^2 + 1/5^2 + ... = (1/1^2 + 1/2^2 + 1/3^2 + ...\_) - (1/2^2 + 1/4^2 + 1/6^2 + ...) = \pi^2/6 - (1/4)(\pi^2/6)$

23. C – $7!/3! = (7)(6)(5)(4) = 840$

24. C – By symmetry, $\sum_{i=0}^{n} \binom{2n+1}{i} = \sum_{i=n+1}^{2n+1} \binom{2n+1}{i} = (1/2)\sum_{i=0}^{2n+1} \binom{2n+1}{i} = (1/2)(2)^{2n+1} = 4^n$, giving $4^{2008} - 4^{2007}$

25. B – The magnitude is $\sqrt{(3)^2 + (4)^2 + (12)^2} = \sqrt{169}$

26. B – $LCM(x,y) \cdot GCF(x,y) = xy$, so $GCF(x,y) = 2688/336 = 8$. (The numbers are 48 and 56).

27. E – There can be two 5's and one 1: 551, 515, 155 work. There can be one 5 and one 1: 51X, 15X, 5X1, 1X5, X51, X15 work. In the first four of these, X can be any digit except 1 or 5 (to avoid double-counting). For the last two of these, X can be any digit except 1, 5, or 0. So there are 3+8+8+8+8+7+7=49 ways.
28. D – The $\arctan$ function is strictly increasing.
29. D – $\dfrac{n^2-2n-3}{n^2-2n+1}=\dfrac{n^2-2n+1}{n^2-2n+1}-\dfrac{4}{n^2-2n+1}=1-\dfrac{4}{(n-1)^2}$ is an integer when $(n-1)^2=4,2,1$. This occurs when $n=3,-1,0,2$

30. B – This path forms a circle of circumference 20 units. So Milton completes the race in 4 seconds and

Astro completes it in 10 seconds.